Title: **The Story of Shape: Extrinsic Characterizations of Immersions**

Abstract: Leonhard Euler believed that one can not define a good curvature measure for surfaces. He wrote in 1763: “la question sur la courbure des surfaces n'est pas susceptible d'une réponse simple, mais elle exige à la fois une infinité de déterminations.'' The quest for the right measure for curvature was settled by C.F. Gauss in 1825, and Sophie Germain introduced the mean curvature in 1831 (her memoir written in 1816 included also the average of principal curvature as a shape invariant). This is the origin of an interesting quest: how do we best quantify the deformation of space? After Berhard Riemann's fundamental contribution from 1854, when he defined what we call today sectional curvature, several other authors have investigated the question of quantifying the deformation of space: Nicolas Renard, Emanoil Bacaloglu, Felice Casorati. The inquiry was pursued well into the 20th century, and it is still of interest today. In 1956 John F. Nash, Jr. proved that a Riemannian manifold can be immersed isometrically into an Euclidean ambient space of dimension sufficiently large, and this major theorem changed the paradigm essentially. A turning point in the history of this question was an enlightening paper written by B.-Y. Chen in 1993, which paved the way for a deeper understanding of the meaning of the Riemannian inequalities between intrinsic and extrinsic quantities. Our present discussion invites a reflection on whether we could hope to characterize submanifolds by using mainly extrinsic quantities. What type of relations can we hope to obtain?

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