Strings, Beams, Approximation Problems, and nonlinear PDEs

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ABSTRACT

This talk will be about the spectrum preserving (isospectral) deformations of classical boundary value problems. I will concentrate on the string equation for which one of the basic deformations leads to the Camassa-Holm equation. If the mass density is a discrete measure one can completely solve this equation with the help of Stieltjes's continued fractions and Padé approximants. If instead one deforms the boundary value problem for the Euler-Bernoulli beam then, using analogous machinery, one is led to a 2-component version of the Camassa-Holm equation. Furthermore, if one chooses two measures appearing in the problem to be discrete measures the solution can be expressed in terms of non-commutative Stieltjes continued fractions. On the PDE side, these families of special solutions to deformation equations corresponding to finite discrete measures are called peakons on account of peaks appearing in their profiles, and they might be interpreted as non-smooth solitons. In the second part of my talk, I will survey some of the decisive developments that shaped my understanding of what peakons are, and my motivation for studying peakon-bearing equations. This talk is in part based on joint work with R. Beals and, independently, with H. Lundmark.