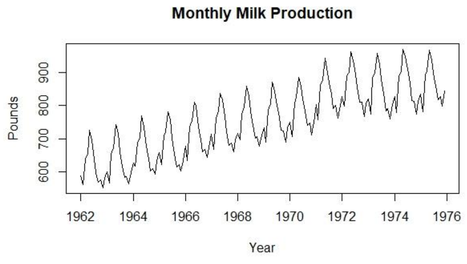
A Time Series Analysis of Dairy Cow Monthly Milk Production

Mai Dao – MATH 5365

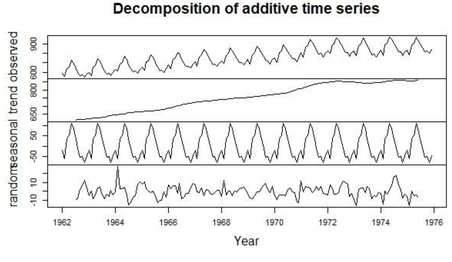
Background and Goals

- Monthly milk production data is from Data Market – Time Series Data Library, measured in pounds per cow.
- Data is taken in England, from January 1962 to December 1975.
- It is of interest for agricultural planning and other related businesses like food and animal production.
- Understand the data's time series characteristics.
- Find the best model that describes the data's behavior.
- Give prediction in a five-month horizon, i.e. five months ahead.

Data and Trend

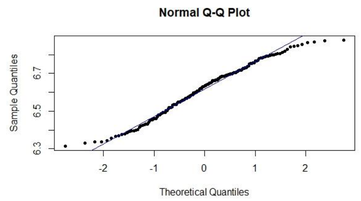


- There is an upward trend, looking somewhat linear.
- Seems to have seasonality within the year.
- Appears to be non-stationary.
- No obvious outliers.
- Decomposition into components for further analysis



Analysis

- Initial analysis involves checking normality and stationarity of the dataset.
- Box-Ljung Test: H_0: Data is independently distributed.
  $p < 2 * 10^{-6} \ll 0.05$, reject H_0.

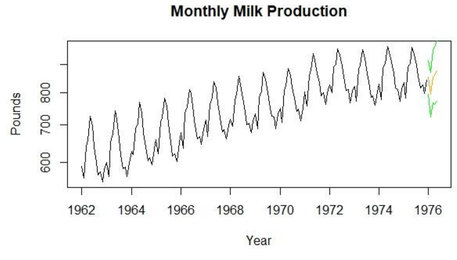


Seasonal ARIMA (log data)

ARIMA$(1,0,1)$x$(0,0,1)_{12}$

$$B^{12}x_t = x_{t-12}$$
$$6.6103 + (1 - 0.8948B)x_t = (1 + 0.0095B)(1 + 0.8850B^{12})e^t$$
$$AIC = -619.34$$
$$RMSE = 0.035$$

Lower 95%	Prediction	Upper 95%
796.20	852.98	913.81
724.37	794.85	872.18
767.02	854.73	952.48
761.77	858.20	966.84
771.53	876.24	995.17

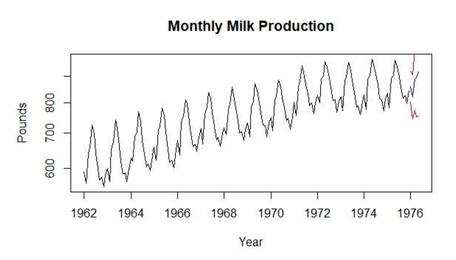


Seasonal ARIMA (differenced log data)

ARIMA$(1,1,3)$x$(0,0,1)_{12}$

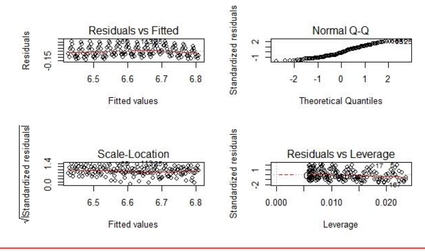
$$(1 + 0.6656B)(1 - B)x_t = (1 + 0.7106B)(1 + 0.3509B)(1 + 0.3635B)(1 + 0.8348B^{12})e^t$$
$$AIC = -626.67$$
$$RMSE = 0.034$$

Lower 95%	Prediction	Upper 95%
802.92	859.58	920.25
743.80	820.89	905.98
774.57	887.09	1015.97
752.99	893.00	1059.04
754.98	918.42	1117.24



Linear Model Fit

$$x_t = -47.46 + 0.0275t$$
$$R^2 = 0.6526$$
$$AIC = -363.30$$



Models

Earlier models considered include:

- Autoregressive model: looks at ACF graph.
  AR(1) Model: $x_t = 0.9397 - 0.2382x_{t-1} + w_t$
  $AIC = 486.78$
- Moving-average model: looks at PACF graph.
  MA(1) Model: $x_t = 0.2275 + w_t + 0.8902w_{t-1}$
  $AIC = 498.90$

Conclusion

- Choose and interpret models based on needs and interest.
- Normally, opt for the simpler model that provides the same amount of information.
- Useful in agricultural planning and econometric analysis.

Interesting Facts

- Dairy cows must give birth to produce milk.
- Most milking dairy cows consume around 100 pounds of feed and 50 gallons of water a day.
- In turn, they can be milked around 3 times daily and can produce around 8 gallons of milk total everyday.
- You can lead cows upstairs not downstairs because their knees cannot bend properly to go downstairs.