Multifractal Detrended Fluctuation Analysis

A Study On Financial Time Series

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- Biomedical signals posses a ***scale invariant*** structure
- A biomedical signal has a scale invariant structure if ***the*** structure ***repeats itself*** on subintervals of the signal.
- The biomedical signal $X(t)$ are scale invariant when

$$X(ct) = c^H X(t)$$

- Fractal analysis estimates the power law exponent, H.
- Monofractal and multifractal structures of the biomedical signal are particular kind of scale invariant structures.
- Monofractal structure of biomedical signals are defined by a single power law exponent
- The scale invariance is independent on time and space in a Monofractal structure

To implement the MFDFA, a MATLAB function is developed by (E. A. F. Ihlen, 2012) as follow:

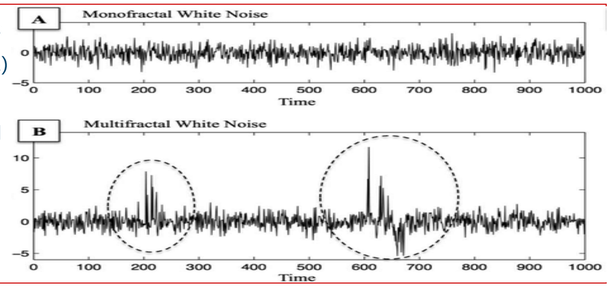
MATLAB function for MFDFA

$$MFDFA1(\text{signal}, \text{scale}, \text{q}, \text{m})$$

Input parameters

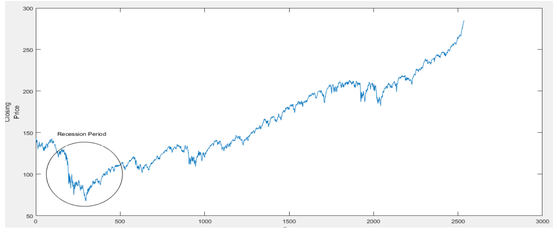
- signal: input signal (time series): In case of having a random walk time series, we should use the differentiated one (Diff(X))
- scale: vector of scales: Segment Sample Size (powers of 2)
- q: q-order weights the local variations and is helpful for differentiating between monofractal and multifractal time series. The difference between monofractal and multifractal time series is that the multifractal time series have local fluctuations with both extreme small and large magnitudes that is absent in the monofractal time series.
- m: The polynomial for the detrending is used to fit to local segments (m= 1,2,3)

- Example of Monofractal VS. Multifractal time series
- Multifractal detrended fluctuation analysis (MFDFA) is a robust analysis used to estimate the multifractal spectrum of power law exponents from a biomedical time series.

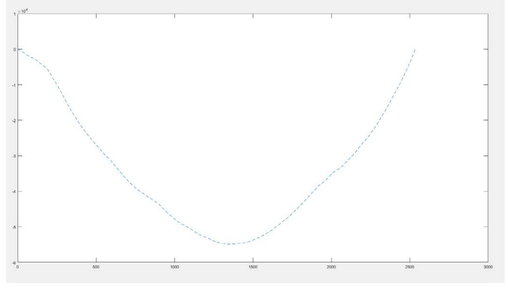


- To differentiate between the magnitudes of the local fluctuations, the ***Root Mean Square (RMS)*** is calculated for segments of time series.
- The slope of the regression line fitted to the plot(scale vs. Overall RMS) gives us the Hurst exponent.
- Case Study: Time series for S&P500

The time series are the closing prices for the index S&P500 for the time period of 2008-2018.

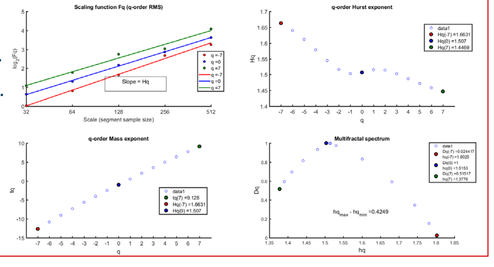


- The time series is converted to a random walk like time series. It is done by subtracting the mean value and integrate the time series.
- The polynomial is then fitted to each segment.
- Finally, the RMS is calculated for separate segments.
- The cumulative sum plot for the S&P500 time series as follow:



- The results of the MFDFA is presented in the following figure.
- The analysis shows that the S&P500 time series are monofractal.

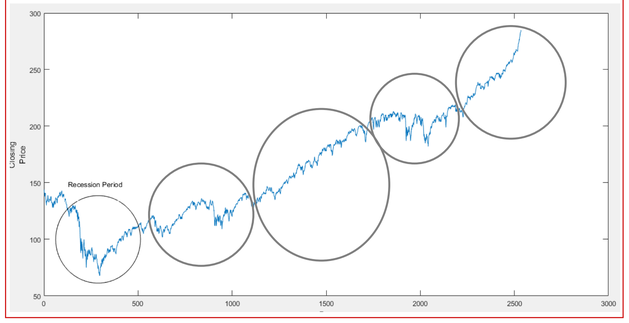
The reason for that is the multifractal spectrum which is less than 1.



- Steps involved in implementing MFDFA:
  1. Random Walk or Noise Like Time Series: The time series are noise like if Hurst exponent H is between0.2 and 0.8. The time series are random walk like when H is between1.2 and1.8. In these cases, the time series should either be differentiated first before implementing any MFDFA
  2. Is the time series scale invariant?
  3. Defining the input parameters

- Future Work

Choosing different sample sizes for different time periods to see the effect on the MFDFA



References:

Ihlen, Espen Alexander Fürst EAFI. "Introduction to multifractal detrended fluctuation analysis in Matlab." Frontiers in physiology 3 (2012): 141.