

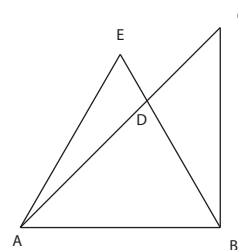
EMMY NOETHER MATHEMATICS DAY  
Texas Tech University  
May 16, 2018

Write your name, the name of your school and your current grade level on the front of the blue book. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

1.) The natural numbers (whole numbers or positive integers) are printed one immediately following the other: 123456789101112131415... . Note that the numbers greater than 9 take up two digits (spaces) to print and the numbers greater than 99 take up three or more digits to print. What is the 1000th digit printed?

2.) The number 12 is equal to exactly four times the sum of its digits ( $12 = 4 \times (1 + 2)$ ). This is also true of the number 24. Determine a positive integer (whole number) which is equal to exactly three times the sum of its digits. Is the number which you have discovered the only one with this property? Either show that this number is unique or discover all such numbers.

3.) Triangle  $\triangle ABE$  is an equilateral triangle, i.e. a triangle where each side has the same length. Triangle  $\triangle ABC$  is an isosceles right triangle, i.e. a triangle with two sides of equal length forming a right angle ( $90^\circ$ ). Each side of triangle  $\triangle ABE$  has length 1. The horizontal and vertical sides of triangle  $\triangle ABC$  each have length 1. Determine the exact area of triangle  $\triangle ADE$ .



4.) A teacher returns a graded exam to her class. The average grade for those who passed the exam was 75. The average grade for those who failed the exam was 45. The average grade for the class was 63. What percentage of students in the class passed the exam?

5.) The ocean liner Gigantic strikes an iceberg and begins taking on water. Water comes into the ship at a constant rate and some amount has already accumulated. The captain determines that 12 identical pumps could pump all of the water out in 3 hours, while five of these pumps could do it in 10 hours. To calm the passengers, the captain wants all of the water out in 2 hours. How many pumps are needed to do this? Note that water is continuing to come into the ship at a constant rate while the pumps are working.

6.) Debra has 12 coins, each of which is either a nickel (5 cent piece), a dime (10 cent piece) or a quarter (25 cent piece). The total value of the coins is \$1.80. Determine how many of each type coin she has, assuming that she has at least one of each type. Show that your answer is the only valid one. (For example, while six nickels and six quarters constitute 12 coins totaling \$1.80, this is not a valid answer because there are no dimes present.)