

EMMY NOETHER MIDDLE SCHOOL MATHEMATICS DAY
Texas Tech University
May 17, 2017

Write your name, the name of your school and your current grade level on the front of the blue book. Work all problems. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

1.) Anne and Betty are planning a three-day canoe trip. Their friend Karen will drop them off at Duncan Landing. From there, they will paddle upstream for 7 hours on the first day. On the second day they will paddle upstream for 8 hours. They have been on this river before and know that their paddling rate is twice the rate of the current in the river. At what time will they have to start downstream on the third day in order to meet Karen at Duncan Landing at 3:00 pm?

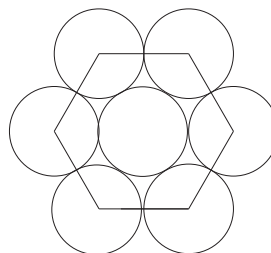
2.) How many three letter words can be formed from the letters A D D I T I O N? A word is any three of the above letters in a specified order, e.g. TDO. A word does not have to have “meaning” or be pronounceable in any particular language.

3.) Do there exist integers (whole numbers) a and b such that $a^2 + b^2 = 2017$? Either determine such integers or show that no such pair of integers exists. Show your reasoning. Do not simply assert an answer.

4.) An analog clock has hour and minute hands that each move at a uniform rate. How many times between 12:00 noon and 12:00 midnight do the hour and minute hands point in exactly opposite directions, 180° apart? Note that 6:00 pm is one such time, with the hour hand pointing exactly at 6 and the minute hand pointing exactly at 12. However, 12:30 pm is not such a time, since at this time the minute hand points exactly at 6 but the hour hand is half way between 12 and 1 and not exactly opposite the minute hand.

Show your reasoning and do not simply assert or guess an answer.

5.) The most efficient way to pack non-overlapping circular discs of the same size in the plane is the hexagon centered arrangement, with a circle centered at each corner of a regular hexagon (six-sided polygon with each side of equal length and each angle equal) and another circle at the center of the hexagon. What fraction of the area of the hexagon is included in the circular discs?



6. Mother’s Day was first celebrated as a national holiday in the United States in 1914. Mother’s Day is always the second Sunday in May. This year Mother’s Day is Sunday May 14. What was the date of the first Mother’s Day? Note that “date” refers to the day of the month of an event, for example May 14, this year, not to the day of the week. Indicate your reasoning and do not simply state (or guess) a day of the week. Remember to allow for leap years.