

EMMY NOETHER HIGH-SCHOOL MATHEMATICS DAY  
Texas Tech University  
May 17, 2017

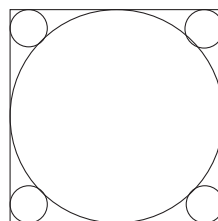
Write your name, the name of your school and your current grade level on the front of the blue book. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

1.) Show that, if  $a$ ,  $b$  and  $c$  are positive integers (whole numbers) such that  $a^2 + b^2 = c^2$ , then at least one of  $a$  or  $b$  is a multiple of 3.

Note that, while it may be useful to consider some examples to see if you can recognize any pattern, you are to show that the above claim is **always** true for any positive integers.

2.) Mother's Day was first celebrated as a national holiday in the United States in 1914. Mother's Day is always the second Sunday in May. This year Mother's Day is Sunday May 14. What was the date of the first Mother's Day? Note that "date" refers to the day of the month of an event, for example May 14, this year, not to the day of the week. Indicate your reasoning and do not simply state (or guess) a day of the week. Remember to allow for leap years.

3.) A circle is inscribed inside a square, tangent to each side of the square, as in the figure at right. In each corner of the square, a smaller circle is inscribed, tangent to the circle and to two sides of the square. The square has each side of length 2. Determine the radius of each smaller circle inscribed in the corner of the square.



4.) How many distinct four-letter words can be formed from the letters M A T H E M A T I C S? A word is an ordered sequence of four letters chosen from the set above, e.g. HMIT. A "word" does not necessarily have to have "meaning" or be pronounceable in any particular "language."

5.) Nancy is walking through a railroad tunnel. She is two-fifths of the way through when she hears a train approaching the tunnel. The train is going a constant 50 miles per hour. If Nancy immediately turns around and runs back to the entrance of the tunnel, she arrives at the entrance just as the train gets there. If instead, she runs directly ahead to the end of the tunnel, she reaches it just as the train reaches that end of the tunnel. Assuming that Nancy runs at the same constant speed in each direction, how fast does she run?

6.) Show that the difference between the square of any two odd numbers is always a multiple of 8. (It is **not** sufficient to find two odd numbers for which this is true. You must show that, if  $m$  and  $n$  are any odd numbers, then  $m^2 - n^2$  is **always** a multiple of 8.)