# EMMY NOETHER MIDDLE SCHOOL MATHEMATICS DAY
Texas Tech University
May 13, 2015

Write your name, the name of your school and your current grade level on the front of the blue book. Work all problems. Show your reasoning and clearly indicate your answer to each problem. Do not simply claim an answer. Partial credit may be given where appropriate. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

1.) A hollow cubical box without a lid contains sixty-four small wooden cubes which fill it exactly. The box is four units long, four units wide and four units high. Each of the small wooden cubes inside it is one unit long, one unit wide and one unit high. How many of these small wooden cubes touch the sides or the bottom of the box?

2.) How many three letter words can be formed from the letters A L G E B R A? A word is any three of the above letters in a specified order, e.g. LBE. A word does not have to have "meaning" or be pronounceable in any particular language.

3.) A regular hexagon (six-sided polygon with each side having the same length and each angle the same) has each side with length one. Determine the area of the hexagon. Give an exact answer with justification, not a decimal approximation.
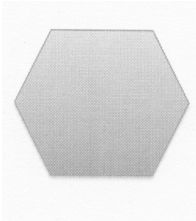
4.) A prime number is a positive integer (whole number) greater than one which is an integer multiple only of itself and one. It is known that there are infinitely many distinct prime numbers. Twin primes are consecutive odd numbers, for example 11 and 13, each of which is prime. It is unknown if there are infinitely many twin prime numbers. Prime triples are three consecutive odd numbers, for example, 3, 5 and 7, each of which is a prime number. Show that 3, 5 and 7 is the only prime triple. Note that you are not simply to show that 3, 5 and 7 are a prime triple but that this is the only set of three consecutive odd numbers that is a prime triple.

5.) The minute and hour hands of an analog clock point in precisely the same direction at 12:00 noon and at 12:00 midnight. How many times between noon and midnight (not including either noon or midnight) do the minute and hour hands of the clock point in exactly the same directions? Clearly indicate your reasoning and do not simply state (or guess) a number. Assume that each hand moves at a uniform rate. (Note that, for example, at 6:30 the hands are not pointing in the same direction since, while the minute hand points directly at 6, the hour hand is halfway between 6 and 7.)

6. The first Earth Day was April 22, 1970. This year Earth Day was on Wednesday April 22, 2015. What day of the week was the first Earth Day? Clearly indicate your reasoning and do not simply state (or guess) a day of the week. Remember to allow for leap years.