EMMY NOETHER HIGH SCHOOL MATHEMATICS DAY
Texas Tech University
May 14, 2014

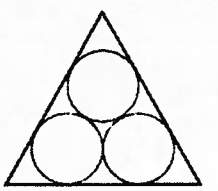
Write your name, the name of your school and your current grade level on the front of the blue book. Work all problems. Show your reasoning and clearly indicate your answer to each problem. Do not simply claim an answer. Partial credit may be given where appropriate. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

1.) A person has an unlimited supply of 5 cent and 11 cent stamps. Show that, by using an appropriate combination of such stamps, every exact postage amount greater than or equal to 40 cents can be formed. (One cannot form 39 cents postage from such stamps.)

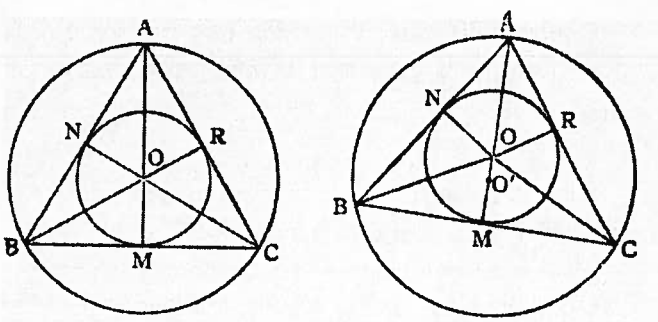
2.) Let $x = 2^{7132}$, $y = 3^{4803}$ and $z = 5^{2998}$. Determine which of x, y or z is the largest and which is the smallest. You must indicate your reasoning to receive credit. Do not merely state (or guess) an answer. (Each of these numbers is too large to directly evaluate using a calculator and it is not assumed that students have access to a calculator. You are not to directly evaluate any of these numbers, but rather to determine their relative size, i.e which is largest and which is smallest.)

3.) There are several times when the hour and minute hands of an analog clock point in exectly opposite directions 180° apart, e.g. at 6:00. An analog clock has hour, minute and second hands. Is there any time when the three hands are equally spaced on the clock face, each 120° away from the others? Either determine a specific time when this happens or show that no such time exists. Assume that each hand moves at a uniform rate. Thus, e.g., 4:40:00 is not such a time since, while the minute hand points directly at 8 and the second hand directly at 12 at this time, the hour hand does not point directly at 4 but is 2/3 of the way from 4 to 5 at this time.

4.) Three identical circles fit tightly inside an equilateral triangle. Each side of the triangle is of length 2. Determine the radius of the largest circle which will fit in the central hole.



5.) One circle is inscribed inside a triangle tangent to each of the sides of the triangle. Another circle is circumscribed outside a triangle passing through the three vertices of the triangle. Prove that the center of the inscribed circle coincides with the center of the circumscribed circle only when the triangle is equilateral.



6.) How many four-letter words can be formed from the letters E M M Y N O E T H E R? A word is any four of the above letters in a specified order, e.g. NMRE. A word does not have to have "meaning" or be pronounceable in any particular language.