EMMY NOETHER HIGH SCHOOL MATHEMATICS DAY
## Texas Tech University
May 15, 2013

Write your name, the name of your school and your current grade level on the front of the blue book. Work all problems. Show your reasoning and clearly indicate your answer to each problem. Do not simply claim an answer. Partial credit may be given where appropriate. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

1.) A point P lies in the same plane as a circle C. The point P is outside of the circle C. The shortest distance from the point P to a point of the circle C is 6 and the greatest distance from the point P to a point of the circle C is 24. Determine the length of the tangent line from the point P to the circle C.

2.) Abraham Lincoln signed the Emancipation Proclamation on January 1, 1863, just over 150 years ago. This year, 2013, January 1 was a Tuesday. On what day of the week did Abraham Lincoln sign the Emancipation Proclamation? Remember to allow for leap years. Also remember that the century year 1900 was not a leap year but the century year 2000 was.

3.) A prime number is an integer (whole number) n greater than 1 which is not an integer multiple of any positive integer other than 1 and n. Each of 3, 5 and 7 is a prime number. Prove that these are the only three consecutive odd integers each of which is prime. (For example 13, 15 and 17 are consecutive odd integers, but 15 is not prime. Note that you are not to give examples but to prove that if x, y and z are consecutive odd integers each of which is prime, then the integers are 3, 5 and 7.)

4.) Four dogs A, B, C and D are initially at the corners of a square meadow. They suddenly begin to pursue each other – A after B, B after C, C after D and D after A. At each moment each dog runs directly toward where the dog he is pursuing is at that time. The dogs are initially 100 meters apart. Each dog runs at a constant speed of 10 meters per second. After what period of time will the dogs meet each other? Will their tracks ever cross, and if so where? What distance does each dog run before meeting the dog he is pursuing? Clearly indicate your reasoning.
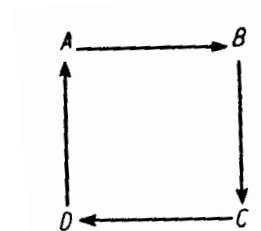
5.) Two squares, each having each edge of length 1, overlap as shown, with the corner of one square at the center of the other. Determine the area of the overlapping region common to the two squares.
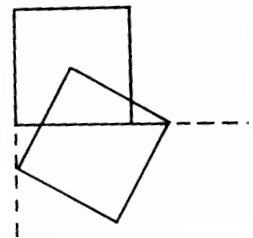
6.) How many four-letter words can be formed from the letters C O N T I N U O U S? A word is any four of the above letters in a specified order, e.g. SNCT. A word does not have to have "meaning" or be pronounceable in any particular language.