

EMMY NOETHER MIDDLE SCHOOL MATHEMATICS DAY  
Texas Tech University  
May 17, 2012

SOLUTIONS.

1.) Going upstream, Anne and Betty are paddling against the current. We are told that they paddle at twice the rate of the current. If the rate of the current is  $c$ , then Anne and Betty paddle at a rate of  $2c$ . Going upstream, their rate relative to the land is  $c - 2c = -c$ . (Subtraction since they are paddling in the direction opposite the current.) Going downstream, their rate relative to the land is  $c + 2c = 3c$ . (Addition since they are paddling with the current.) Thus, relative to the land Anne and Betty travel downstream three times as fast as they travel upstream. On the first two days, they travel upstream for a total of 15 hours, 7 hours the first day and 8 hours the second day. Thus, it will take them one-third as long, or 5 hours to travel the same distance downstream. They must start downstream on the third day 5 hours before they are to meet Karen at 3:00 pm, hence by 10:00am.

2.) There are two cases to consider, whether all three letters of the word are different or whether the word contains both letters A. If all three letters are different, then there are six choices for the first letter of the word, five choices for the second letter and four choices for the third letter, or  $6 \cdot 5 \cdot 4 = 120$  possible words. If the word includes both letters A, then there are three choices for which spaces the A's occur in the word (first and second letter, first and third letter or second and third letter) and for each of these choices five choices for the remaining letter, or  $3 \cdot 5 = 15$  possible words. There is thus a total of  $120 + 15 =$ 135 possible words.

3.) If the hour is 1 through 5, since only three digits are displayed, one for the hour and two for the minutes, they must all be the same, i.e. 1:11, 2:22, 3:33, 4:44 or 5:55, for five possibilities. If the hour is 6 through 9, there is no valid time display with all three digits the same, since 6:66, 7:77, 8:88 or 9:99 are not valid times. If the hour is 10, then the minutes must be 00 or 11, for times of 10:00 or 10:11 and two possible times. If the hour is 11, the the minutes can be any time containing at least one 1, i.e the time can be 11:01, any time 11:10 through 11:19, 11:21, 11:31, 11:41 or 11:51 for 15 possible times. If the hour is 12, then the minutes must be 11 or 22, for times of 12:11 and 12:22. There is thus a total of  $5 + 2 + 15 + 2 =$ 24 possible times.

4.) There is a total of nine puppies in the two litters combined. We are told that four of these are sable and white and that every puppy in either litter is either sable and white or tricolor. Thus, there are five tricolor puppies in the two litters combined. Each litter has exactly one tricolor female puppy. Thus there are 3 tricolor male puppies in the two litters combined.

5.) We are not given the dimensions of the circles or the hexagon. However, we are to determine the fraction of the area of the hexagon included in the circular discs. This ratio is independent of scale. For convenience, assume that each circle has radius 1. Each side of the hexagon consists of the radii of two adjacent circles and so is of length 2. Each circle has area  $\pi \cdot (1)^2 = \pi$ . Inside the hexagon is the area of the central circle and  $1/3$  the area

of each of the outer circles. Thus, the area of the hexagon included in the circular discs is  $1 \cdot \pi + 6 \cdot \frac{1}{3}\pi = 3\pi$ . The hexagon can be divided into six equilateral triangles, each having base along one side of the hexagon and opposite vertex at the center of the central circle. The area of each such triangle is  $\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2)(\sqrt{3}) = \sqrt{3}$ . Thus the area of the hexagon is  $6 \cdot \sqrt{3}$ . The fraction of the area of the hexagon included in the circular discs is thus  $\frac{3\pi}{6\sqrt{3}} \approx 0.9069$ .

6.) Let  $n$  be the number of nickels,  $d$  the number of dimes and  $q$  be the number of quarters which Kay has. Thus  $n + d + q = 10$  and  $5n + 10d + 25q = 85$ . Just from the two equations in three variables, we cannot uniquely solve for  $n$ ,  $d$  or  $q$ . However, we also know that each of  $n$ ,  $d$  and  $q$  must be a positive integer. If  $q = 2$ , then there is 50 cents in quarters, leaving 35 cents for the remaining eight coins. This is not possible with each coin being worth at least five cents. If there are three quarters, the disparity is even greater. Thus, there must only be one quarter. There is then 60 cents in a combination of 9 nickels and dimes. This can only occur with 6 nickels and 3 dimes. Thus, there must be  $\boxed{6 \text{ nickels, } 3 \text{ dimes and } 1 \text{ quarter.}}$