EMMY NOETHER HIGH SCHOOL MATHEMATICS DAY
Texas Tech University
May 17, 2012

Write your name, the name of your school and your current grade level on the front of the blue book. Work all problems. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

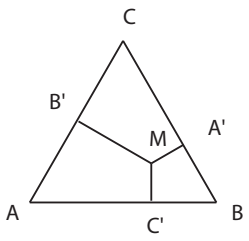
1.) Prove that if n is an odd integer (whole number) then n^2-1 is a multiple of 8. (Note that you are to prove that this is true for *every* odd integer, not just give examples for particular odd integers. It may be useful to note that an odd integer can be expressed in the form $n = 2k + 1$ for some integer k.)

2.) April 13 of this year was on a Friday. Friday 13 is traditionally considered "unlucky." Show that every calendar year has at least one Friday 13 and that a calendar year can have a maximum of three Friday 13's.

3.) A teacher returns a graded exam to her class. The average grade for those who passed the exam was 78. The average grade for those who failed the exam was 38. The average grade for the entire class was 64. What percentage of students in the class passed the exam?

4.) The ocean liner Gigantic strikes an iceberg and begins taking on water. Water come into the ship at a constant rate and some amount has already accumulated. The captain determines that 10 identical pumps could pump all of the water out in 4 hours, while 6 of these pumps could do it in 12 hours. To calm the passengers, the captain wants all of the water out in 2 hours. How many pumps are needed to do this? Note that water is continuing to come into the ship at a constant rate while the pumps are working.

5.) Let $\triangle ABC$ be an equilateral triangle and let M be a point in the interior of the triangle. Let MA', MB' and MC' be the lengths of the perpendiculars from the point M to the sides BC, AC and AB respectively (opposite vertices A, B and C respectively). Show that the sum $MA' + MB' + MC'$ is a constant, independent of the choice of the point M.



6.) How many four letter words can be formed from the letters C A L C U L U S? A word is any four of the above letters in a specified order, e.g. SLCC. A word does not have to have "meaning" or be pronounceable in any particular language.