EMMY NOETHER MIDDLE SCHOOL MATHEMATICS DAY
Texas Tech University
May 11, 2011

Clearly write your name, the name of your school and your current grade level on the front of the blue book. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. You should justify your answer in order to receive full credit. It is possible to receive partial credit. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover. There are prizes for the best score from each school as well as for the best score at each grade level.

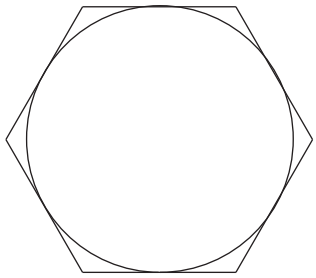
1.) At 12:00 noon and at 12:00 midnight the hour hand and the minute hand on an analog clock point in the same direction. At how many other times between noon and midnight do the hour hand and the minute hand of an analog clock point in the same direction? Do not count noon or midnight. Justify your answer.

2.) The number 12 is exactly four times the sum of its digits. ($12 = 4(1+2)$) The number 18 is exactly twice the sum of its digits. ($18 = 2(1+8)$) Determine a positive integer (whole number) which is exactly three times the sum of its digits? Is your answer the only one? Either show that there are other such numbers or show that your answer is unique.

3.) Determine how many three letter words can be formed from the letters
M A T H E M A T I C S.
A word is a string of three letters in a specific order, e.g. HST. A word does not have to have "meaning" in any particular language. Justify your answer.

4.) Do there exist positive integers (whole numbers) a and b such that $a^2 + b^2 = 2011$? Either determine such numbers a and b or prove that no such numbers exist.

5.) A regular hexagon (six-sided figure with each side of the same length and with all angles equal) is circumscribed about a circle of radius 1. Determine the area of the hexagon. Give an exact answer, not a decimal approximation.



6.) This April marked the 50th anniversary of the first human spaceflight. Yuri Gagarin orbited the Earth on April 12, 1961. April 12 of this year (2011) was a Tuesday. On what day of the week did Yuri Gagarin become the first human to orbit the Earth? Justify your answer. Do not simply guess or assert an answer. Remember to allow for leap years.