EMMY NOETHER HIGH-SCHOOL MATHEMATICS DAY
Texas Tech University
May 11, 2011

Clearly write your name, the name of your school and your current grade level on the front of the blue book. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. You should justify your answer in order to receive full credit. It is possible to receive partial credit. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover. There are prizes for the best score from each school as well as for the best score at each grade level.

1.) The hour hand of an analog clock is between 10 and 11 on the clock dial. The minute hand is exactly opposite the hour hand, 180° away. Determine the exact time. Do not approximate to the nearest minute. Give any appropriate fraction of minutes. Assume that both hands move at a uniform rate. For example, at 10:15 the hour hand hand is 1/4 (15/60) of the way from 10 to 11.

2.) The number 12 is four times the sum of its digits, i.e. $12 = 4(1+2)$ and the number 18 is twice the sum of its digits, i.e. $18 = 2(1+9)$. The number 12 is not unique with this property, since there are other positive integers (whole numbers) which are also four times the sum of their digits. There is only one positive integer which is five times the sum of its digits. Determine this number and prove that it is the only positive integer with this property.

3.) Determine how many five-letter words can be formed from the letters
   M A T H E M A T I C S.
   A word is a sequence of five letters in a specific order, e.g. HSTMM. A word does not have to have "meaning" in any specific language. Justify your answer.

4.) A prime number is a positive integer (whole number) greater than one which is divisible only by itself and one. For example, 2, 3, 5, 7 and 11 are prime numbers. A perfect kth power is a positive integer of the form n^k for some positive integer n. For example, $25 = 5^2$ is a perfect square and $8 = 2^3$ is a perfect cube. 3 is a prime number which is one less than a perfect square and 7 is a prime number which is one less than a perfect cube. Prove that there is no prime number which is one less than a perfect fourth power.

5.) This April marked the 150th anniversary of the beginning of the American Civil War. The American Civil War began with the firing on Fort Sumter on April 12, 1861. This year (2011) April 12 was a Tuesday. On what day of the week did the American Civil War begin? Remember to allow for leap years. Remember also that the century year 1900 was not a leap year while the century year 2000 was. Justify your answer. Do not simply guess or assert an answer.

6.) An equilateral triangle (triangle with each side of the same length) is circumscribed about a square as in the figure at the right. Each side of the square has length 1. Determine the area of the triangle. Justify your answer. Give an exact answer, not an approximation.