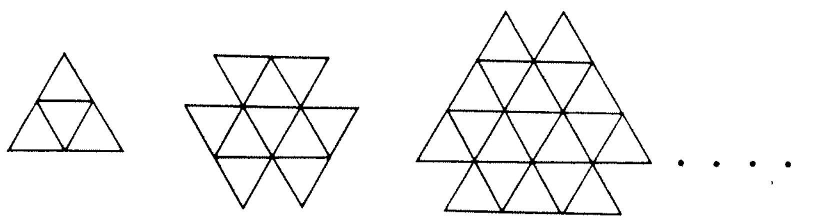
EMMY NOETHER HIGH-SCHOOL MATHEMATICS DAY
Texas Tech University
May 12, 2010

Write your name, the name of your school and your current grade level on the front of the blue book. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. You should justify your answer in order to receive full credit. It is possible to receive partial credit. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover. There are prizes for the best score from each school as well as for the best score at each grade level.

1.) The positive odd integers (whole numbers) are written one immediately after the other: 135791113151719 21.... Determine the 250th digit in this listing. A digit is one of the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 or 0. For example the tenth digit in this listing is the "1" in "15."

2.) Start with an equilateral triangle (a triangle with all three sides of equal length). At the first stage add new equilateral triangles of the same size along each edge. At each successive stage add new equilateral triangles of the same size along each outside edge. See the figures below for reference. How many small triangles are there by the time that triangles have been added at the tenth stage?



3.) The planet Mercury rotates on its axis in 58.6 days. It revolves about the Sun in 87.9 days. (Notice that its period of revolution is precisely 1.5 times its period of rotation.) Its rotation and its revolution are in the same direction. Determine the length of the solar day on Mercury, i.e. for a given spot on the surface of Mercury how long is it between when the Sun passes overhead at local noon and the next time the Sun passes overhead?

4.) A prime number is a positive integer (whole number) greater than one which is divisible only by itself and one. A perfect square is a positive integer which is equal to the product of a positive integer x with itself, i.e. $x \times x$. Thus 3 is a prime number and 4 is a perfect square ($4 = 2 \times 2$). Prove that 3 is the only prime number which is one less than a perfect square.

5.) One has an unlimited number of 3 cent and 5 cent stamps. Show that by using just 3 cent and 5 cent stamps one can form every exact postage amount greater than 7 cents. (Note, that doing this for several amounts from 8 cents up may *suggest* that this can always be done, but such alone does not *show* that it can **always** be done. One needs an argument that it can **always** be done, not just several examples.)

6.) A square is inscribed inside a circle of radius 1. In each of the four regions between the square and the circle a smaller square is inscribed. See the accompanying figure. Determine the length of each side of each of the smaller squares.