

EMMY NOETHER MIDDLE SCHOOL MATHEMATICS DAY
Texas Tech University
May 12, 2010

SOLUTIONS.

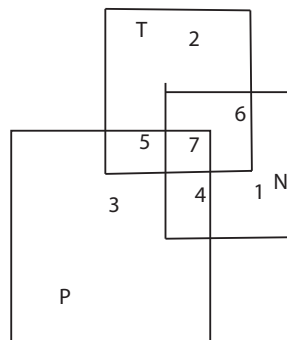
1.) The two diagonals of the square room have the same length and the same number of tiles covering them. Since there is an odd number of tiles covering the two diagonals, they have a tile in common at the center of the room. Since there are 49 tiles covering the diagonals, each diagonal is covered by 25 tiles, with the center tile in common. The tiles covering a diagonal meet corner to corner and are arranged with the same orientation, so there are also 25 tiles along each edge of the room. Since each tile is one foot on a side, the area of the room is $25 \times 25 = \boxed{625}$ square feet.

2.) Any one digit number is equal to the sum of its digits. Any number from 10 through 13 has the sum of its digits at most 4 and so is not equal to twice this sum. Any number from 20 to 29 has sum of its digits at most 11 and so twice the sum of its digits at most 22. Any number from 30 to 39 has sum of its digits at most 12 and so twice the sum of its digits is at most 24. Any number from 40 through 99 has sum of its digits at most 18 and so twice the sum of its digits is at most 36. Any number with three or more digits is more than twice the sum of its digits. Thus, the only positive integers to check are 14 through 22. A quick check shows that $\boxed{18}$ is the $\boxed{\text{only}}$ positive integer which is equal to twice the sum of its digits.

3.) A newspaper is folded so that when opened a left inside page is even numbered and a right inside is odd numbered, with left outside being odd numbered and right outside being even numbered. Left pages are lower numbered than right pages. Thus page 6 is on the left inside and page 19 is on the right inside, with pages 5 and 20 printed on the same sheet. With this same arrangement, pages 1 and 24 would be printed on the left outside and right outside of a single sheet and would constitute the first and last page of the newspaper. Thus the newspaper has $\boxed{24}$ pages.

4.) Consider the three sets P of puppies with white paws, T or puppies with white tips to their tails and N with white around their necks. Possible intersections of these sets is illustrated at right. Using the information given and starting at the intersection $P \cap T \cap N$ and working out, one has the indicated number of puppies in each of the portions of the sets. The total is thus $\boxed{28}$ puppies.

(This color pattern was chosen since collie puppies always have white paws, white tips to their tails and white around their necks, whatever color they may be elsewhere.)



5.) Since the inner square has area 16, each side of it has length 4. Its diagonal, which is also a diameter of the circle, thus has length $4\sqrt{2}$. The length of the diameter of the circle is also the length of each side of the outer square and so the outer square has area $4\sqrt{2} \times 4\sqrt{2} = \boxed{32}$ and the circle has radius half its diameter or $\boxed{2\sqrt{2}}$. Alternatively, if one rotates the inner square 45° so that its corners touch the sides of the larger square, one sees that the larger square has twice the area of the inner square, agreeing with the computation above.

6.) Each non-leap year has 365 days, or 52 weeks of 7 days each ($52 \cdot 7 = 364$) plus one extra day. Each leap year has 366 days, or 52 weeks of 7 days plus two extra days. In the 40 years since the Apollo 13 explosion on the way to the moon there have been 30 non-leap years and 10 leap years, so there have been $30 \cdot 1 + 10 \cdot 2 = 50$ extra days, besides the exact number of weeks. However, 50 days is equivalent to 7 weeks ($7 \cdot 7 = 49$) plus one extra day. Thus the day of the week on the 40th anniversary this year is one week day later than the initial day of the explosion and the Apollo 13 spacecraft explosion occurred on $\boxed{\text{Monday April 13, 1970.}}$