EMMY NOETHER MIDDLE SCHOOL MATHEMATICS DAY
Texas Tech University
May 6, 2009

SOLUTIONS.

1.) If the cost to the store of the shoes is denoted by C, then the store has a marked price of 1.20 C for the shoes. The customer buys these for 15% off marked price or for $(0.85)(1.20\ C) = 1.02\ C$. This is $1.00 more than the cost to the store, so $1.02\ C = C + \$1.00$. Hence $0.02\ C = \$1.00$ and $C = \$50.00$. The customer thus buys the shoes for $1.02\ C = C + \$1.00 = \boxed{\$51.00.}$

2.) If we denote the number of pennies by P, the number of dimes by D and the number of quarters by Q, then we have that $P + D + Q = 13$ and $1P + 10D + 25Q = 97$. However, there are infinitely many combinations of numbers which satisfy these two equations. We need to determine a solution where each of P, D and Q is a nonnegative integer (whole number). Any combination of dimes and quarters has a value which is a multiple of 5 cents. Since the total value of the coins is two more than a multiple of 5, the number of pennies must be 2, 7 or 12. If there are only 2 pennies, then there is a total of 11 dimes and quarters with a combined value of $0.95. This is impossible, since eleven coins each worth ten cents or more cannot have a combined value less than a dollar. If there are 7 pennies, then there is a total of 6 dimes and quarters with a combined value of $0.90. This happens with four dimes and two quarters. If there are 11 pennies, then is only one dime or quarter but a value of $0.85 in dimes or quarters, and impossibility. Thus Susan has $\boxed{\text{7 pennies, 4 dimes and 2 quarters.}}$

3.) There are three cases to consider. Either the three letter word has two T's and a different letter, it has two E's and a different letter or it has three different letters. There are a total of seven different letters. If the word has two T's, there are three possible pairs for which letters are T's (the first and second, the first and third or the second and third). For each of these possibilites, there are six possibilities as to what the other letter is, giving $3 \cdot 6 = 18$ three letter words having two T's. Similarly, there are $3 \cdot 6 = 18$ three letter words having two E's. If all three letters are different, there are seven possibilities for the first letter, six possibilities for the second letter (since it must be different from the first) and five possibilities for the third letter (since it must be different from the first and second), giving $7 \cdot 6 \cdot 5 = 210$ three letter words with all letters distinct. There are thus a total of $18 + 18 + 210 = \boxed{246}$ distinct three letter words formed from the given letters.

4.) The minute hand of a clock makes a complete circle of $360°$ degrees in one hour (60 minutes). Thus, it moves $360°/60 = 6°$ degrees per minute. At 24 minutes after the hour it has moved $24 \cdot 6° = 144°$ from its position straight up at 12 on the clock face. The hour hand of a clock makes a complete circle of $360°$ in 12 hours (720 minutes). Thus it moves $360°/720 = (1/2)°$ per minute. At 1:24 (or 84 minutes after 12:00) it has moved $84 \cdot (1/2)° = 42°$ from its position straight up at 12 on the clock face. The angle from the hour hand clockwise to the minute hand is thus $144° - 42° = \boxed{102°.}$

5.) Consider the two triangles, $\triangle BCD$ and $\triangle ABE$. Each contains a right angle, $\triangle BCD$ having a right angle at D and $\triangle ABE$ having a right angle at E. Each triangle also includes the angle at B. Thus two angles of $\triangle ABC$ are equal to two angles of $\triangle ABE$. Since the sum of the angles of a triangle is a constant, the third angles of these two triangles must also be equal and $\boxed{\angle EAB = \angle BCD.}$

6.) Each non-leap year has 365 days, or 52 weeks of 7 days each ($52 \cdot 7 = 364$) plus one extra day. Each leap year has 366 days, or 52 weeks of 7 days plus two extra days. In the 40 years since the Apollo 11 landing on the moon there have been 30 non-leap years and 10 leap years, so there have been $30 \cdot 1 + 10 \cdot 2 = 50$ extra days, besides the exact number of weeks. However, 50 days is equivalent to 7 weeks ($7 \cdot 7 = 49$) plus one extra day. Thus the day of the week on the 40th anniversary this year is one week day later than the initial landing and the Apollo 11 spacecraft landed on the Moon on $\boxed{\text{Sunday July 20, 1969.}}$