# EMMY NOETHER HIGH-SCHOOL MATHEMATICS DAY
Texas Tech University
May 6, 2009

Write your name, the name of your school and your current grade level on the front of the blue book. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover. There are prizes for the best score from each school as well as for the best score at each grade level.

1.) Show that the difference between the squares of two odd numbers is always a multiple of 8. (It is not sufficient to find two odd numbers for which this is true. You must show that this is always true no matter what odd numbers are chosen.)

2.) Every integer (whole number) can be expressed in the form $5 \cdot x + 7 \cdot y$ where x and y are also integers. For example $1 = 5 \cdot (-4) + 7 \cdot 3$ and $2 = 5 \cdot 13 + 7 \cdot (-9)$. However, in each of these two cases one of x or y must be negative. Every sufficiently large integer can be expressed as $5 \cdot x + 7 \cdot y$ where both x and y are nonnegative. For example $1001 = 5 \cdot 105 + 7 \cdot 68$. Determine the largest integer which cannot be expressed in the form $5 \cdot x + 7 \cdot y$ with both x and y nonnegative.

3.) A rectangle is divided into three regions of equal areas by lines drawn from the upper right corner to the bottom and left sides of the rectangle. See the figure at the right. Determine the ratios into which the intersections of these lines with the bottom and left sides of the rectangle divide the bottom and left sides. Justify your answer, do not simply assert or guess a ratio. Determine whether the ratio on the bottom side is the same as the ratio on the left side and whether these ratios depend on the lengths of the sides of the rectangle or on the relative proportions of these sides.
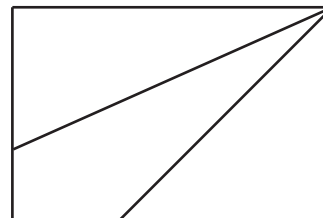
4.) The hands on a clock move at a uniform rate. For example, at 4:30 the hour hand is exactly half way between 4 and 5 on the clock dial. A short while later the hour hand is still between 4 and 5 on the dial and the minute hand is exactly $180°$ away from the hour hand. Determine the exact time when this happens. Remember that there are $360°$ in a complete circle, so at the indicated time the hour and minute hands point in exactly opposite directions.

5.) Abraham Lincoln and Charles Darwin were both born on February 12, 1809. Their 200th birthday occurred this year on Thursday February 12, 2009. Determine on what day of the week Abraham Lincoln and Charles Darwin were born. You must indicate reasoning (not just a guess or claimed answer) in order to receive full credit. Remember that years divisible by 4 are leap years, with the exception that century years are leap years only if divisible by 400 (i.e. 1900 was not a leap year but 2000 was, as were other leap years from 1812 through 2008).

6.) Alice intends to withdraw x dollars and y cents from her bank account. After spending exactly \$1.00, she discovers that she mistakenly withdrew y dollars and x cents from the account and now has exactly half as much money as she intended to withdraw. She did not have any cash immediately prior to this withdrawal. How much money did Alice intend to withdraw?