

EMMY NOETHER MIDDLE SCHOOL
MATHEMATICS DAY
Texas Tech University
May 8, 2008

SOLUTIONS.

1.) If we denote Rachel's favorite number by n , then, since we are told that dividing n by 7 gives a remainder of 5, we know that n is of the form $n = 7x + 5$ for some integer (whole number) x . Multiplying this by 3 gives $3n = 3 \cdot (7x + 5) = 21x + 15$. Dividing this by 7 gives $(21x + 15) = (3x + 2) \cdot 7 + 1$. Thus the remainder is $\boxed{1}$.

2.) We are not told the value (n cents) of the coins with which Susan starts. However, after giving a clerk 55 cents and receiving a penny back she has 51 cents. Thus $n - 55 + 1 = 51$ or $n = 105$. If x is the number of nickels with which she starts and y is the number of quarters with which she starts, then we have that $x + y = 9$ and $5x + 25y = 105$. Solving this system of two equations gives that $x = 6$ and $y = 3$. Thus Susan started with $\boxed{6 \text{ nickels and } 3 \text{ quarters}}$.

3.) There are two cases to consider. Either the three letter word uses both of the E's or it does not. In the first case, there are three choices for which positions in the three letter word are occupied by E's (which of the three positions is not an E) and for each of these choices there are five choices for the remaining letter. Hence there are $3 \cdot 5 = 15$ three letter words containing two E's. In the second case, all three letters of the word are distinct. There are six choices for the first letter, five choices for the second letter and four choices for the last letter (since each letter must be different from the previous ones). There are thus a total of $6 \cdot 5 \cdot 4 = 120$ three letter words with all letters distinct. There are thus a total of $15 + 120 = \boxed{135}$ distinct three letter words formed from the given letters.

4.) The minute hand of a clock makes a complete circle of 360° degrees in one hour (60 minutes). Thus, it moves $360^\circ/60 = 6^\circ$ degrees per minute. At 16 minutes after the hour it has moved $16 \cdot 6^\circ = 96^\circ$ from its position straight up at 12 on the clock face. The hour hand of a clock makes a complete circle of 360° in 12 hours. Thus it moves $360^\circ/12 = 30^\circ$ per hour. At 10:16 it has moved $(10 + \frac{16}{60}) \cdot 30^\circ = 308^\circ$ from its position straight up at 12 on the clock face. The angle from the minute hand clockwise to the hour hand is thus $308^\circ - 96^\circ = 212^\circ$. The smaller angle, from the hour hand clockwise to the minute hand is $360^\circ - 212^\circ = \boxed{148^\circ}$.

5.) Each side of the triangle is the union of radii of two of the circles. Thus the sides of the triangle are of lengths $6(= 2 + 4)$, $8(= 2 + 6)$ and $10(= 4 + 6)$. Since $6^2 + 8^2 = 10^2$, the triangle is a right triangle with legs of length 6 and 8 and hypotenuse of length 10. The area of the triangle is thus $\frac{1}{2} \cdot 6 \cdot 8 = \boxed{24}$.

6.) There are a total of 11 puppies in the two litters (six puppies in the first litter and five puppies in the second litter). There are five tricolor puppies, since every puppy is either sable and white or tricolor and there are six sable and white puppies in the two litters combined. There are two female tricolor puppies, one in each litter. Hence, there are $\boxed{3}$ tricolor male puppies in the two litters combined. (One can see a tricolor male collie at <http://www.math.ttu.edu/~wlewis/crockett.html>.)