EMMY NOETHER HIGH-SCHOOL MATHEMATICS DAY
Texas Tech University
May 8, 2008

Write your name, the name of your school and your current grade level on the front of the blue book. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

1.) Determine which of the following numbers is smallest and which is largest. Justify your answer. You must show reasoning (not just a guess or a claimed answer) in order to receive full credit.

$$2^{3543} \qquad 3^{2776} \qquad 5^{1836}$$

2.) On a given date Venus is on a line between the Earth and the Sun. It takes Earth 365 days to go around the Sun and it takes Venus 225 days to go around the Sun. When will Venus next be on a line between the Earth and the Sun? Assume that Venus and the Earth go around the Sun in the same direction, each moving at a constant angular rate.

3.) A circle is inscribed inside a square, tangent to each side of the square, as in the figure at right. In each corner of the square, a smaller circle is inscribed, tangent to the circle and to two sides of the square. The square has each side of length 2. Determine the radius of each smaller circle inscribed in the corner of the square.
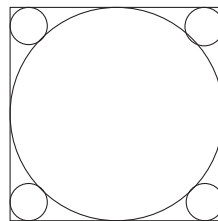
4.) How many distinct five-letter words can be formed from the letters E M M Y N O E T H E R? A word is an ordered sequence of five letters chosen from the set above, e.g. RNEME. A "word" does not necessarily have to have "meaning" in any particular "language."

5.) Nancy is walking through a railroad tunnel. She is two-fifths of the way through when she hears a train approaching the tunnel. The train is going a constant 50 miles per hour. If Nancy immediately turns around and runs back to the entrance of the tunnel, she arrives at the entrance just as the train gets there. If instead, she runs directly ahead to the end of the tunnel, she reaches it just as the train reaches that end of the tunnel. Assuming that Nancy runs at the same constant speed in each direction, how fast does she run?

6.) Debra has 12 coins, each of which is either a nickel (5 cent piece), a dime (10 cent piece) or a quarter (25 cent piece). The total value of the coins is $1.80. Determine how many of each type coin she has, assuming that she has at least one of each type. Show that your answer is the only valid one. (For example, while six nickels and six quarters constitute 12 coins totaling $1.80, this is not a valid answer because there are no dimes present.)