

EMMY NOETHER JUNIOR HIGH-SCHOOL MATHEMATICS DAY
Texas Tech University
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SOLUTIONS.

1.) If one tries directly to solve the given equations algebraically, one has complicated computations. However, one can determine a solution without algebra. If each of x and y is assumed to be an integer (whole number), then each must also be a perfect square (since both \sqrt{y} and \sqrt{x} must be integers). Thus from the first equation one has that $x \leq 25$ (the largest perfect square less than or equal to 32) and hence $\sqrt{y} \geq 7$. From the second equation one has that $y \leq 49$ (the largest perfect square less than or equal to 54) and hence $\sqrt{x} \geq 5$. Thus the solution is $x = 25$ and $y = 49$.

2.) One is given that Nancy runs at the same constant speed in either direction. If she immediately runs back to the entrance of the tunnel, she gets there just as the train does. If instead, she runs for the same length of time (and hence same distance) towards the end of the tunnel, she is two-thirds of the way through the tunnel (the original one-third of the way through she had walked before hearing the train and the one-third distance she has just run) when the train arrives at the entrance. She then runs the remaining one-third of the length of the tunnel, arriving at the tunnel's end just as the train also reaches the end of the tunnel. Thus, Nancy runs one-third the length of the tunnel while the train travels the entire length of the tunnel. Hence Nancy runs one-third as fast as the train, or 10 miles per hour.

3.) The ones digits will be used up before any of the other digits. For the houses numbered 1 through 99, there will be 20 ones digits used, ten in the ones position for 1, 11, 21, through 91 and 10 in the tens position for 10, 11, 12, through 19. For the houses numbered 100 through 109, there will be 11 ones digits used, one in the ones position for 101 and ten in the hundreds position for 100, 101, 102 through 109. For the houses numbered 110 through 119, there will be 21 ones digits used, one for the ones position in 111 and ten in each of the tens and hundreds positions for each of these numbers. So far, for the numbers 1 through 119, one has used 52 ones digits ($20 + 11 + 21$). For each of the sets of ten houses numbered 120 through 129, 130 through 139, 140 through 149 and 150 through 159, one uses 11 ones digits, one in the ones position and ten in the hundreds positions (one for each of the numbers in the set). Thus, through house number 159 one uses 96 ones digits ($52 + 11 + 11 + 11 + 11$). One then uses 1 one digit for house 160, 2 ones digits for house 161 and 1 one digit for house 162. At this point one has used up all of the 100 ones digits and there is none available to be used for house 163 .

4.) Going upstream, Patty and Debra are paddling against the current. We are told that they paddle at twice the rate of the current. If the rate of the current is c , then Patty and Debra paddle at a rate of $2c$. Going upstream, their rate relative to the land is $c - 2c = -c$. (Subtraction since they are paddling in the direction opposite the current.) Going downstream, their rate relative to the land is $c + 2c = 3c$. (Addition since they are paddling with the current.) Thus, relative to the land Patty and Debra travel downstream three times as fast as they travel upstream. On the first two days, they travel upstream for a total of 21 hours, 12 hours the first day and 9 hours the second day. Thus, it will take them one-third as long, or 7 hours to travel the same distance downstream. They must start downstream on the third day 7 hours before they are to meet Marge at 5:00 pm, hence by $10:00$ am.

5.) The sum of the digits is an integer (whole number) and the number itself is 13 times as large as the sum of its digits. Hence the number must be a multiple of 13. One can quickly consider the small multiples of 13. The number 13 has sum of digits 4, but is not 4×13 . The number 26 has sum of digits 8, but is not 8×13 . Considering the next several multiples of 13, one quickly finds that the sum of digits of 117 is 9 and $117 = 13 \times 9$ is the smallest such number.

6.) An expression of the form $xx/yy/07$ has meaning in both systems if and only if each of xx and yy is a number from 1 through 12. However, if xx and yy are the same, as in $5/5/07$, both date systems give the same meaning (May 5, 2007 or 5 May 2007 in this case) and so the date is not ambiguous. There are thus ambiguous expressions with 12 possible values for xx and 11 possible values for yy for each xx (since yy must be different from xx). During the year there are a total of $132 = 12 \times 11$ expressions which are ambiguous, having distinct date interpretations in the two systems.