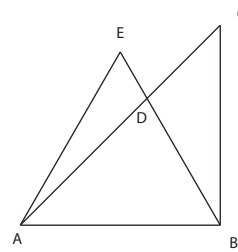


EMMY NOETHER HIGH-SCHOOL MATHEMATICS DAY
Texas Tech University
May 9, 2007

Write your name, the name of your school and your current grade level on the front of the blue book. Show your reasoning and clearly indicate your answer to each problem. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

- 1.) Determine the value of x if $\frac{1}{27} \cdot 3^{100} \cdot \frac{1}{81} \cdot 9^x = \frac{1}{3} \cdot 3^x$.
- 2.) At 12:00 noon the hour hand, minute hand and second hand of a clock are all pointing straight upward.
 - a.) Find the first time after 12:00 noon at which the angle between the hour hand and minute hand is 120° .
 - b.) Is there a time when the three hands divide the face of the clock into equal thirds, i.e. the angles between the hour hand, minute hand and second hand are each equal to 120° ?
Remember that a full circle has 360° , so 120° is one-third of the way around a circle.

3.) Triangle ABE is an equilateral triangle, i.e. a triangle where each side has the same length. Triangle ABC is an isosceles right triangle, i.e. a triangle with two sides of equal length forming a right angle (90°). Each side of triangle ABE has length 1. The horizontal and vertical sides of triangle ABC each have length 1. Determine the exact area of triangle ADE.



- 4.) A teacher returns a graded exam to her class. The average grade for those who passed the exam was 65. The average grade for those who failed the exam was 35. The average grade for the entire class was 53. What percentage of students in the class passed the exam?
- 5.) The ocean liner Gigantic strikes an iceberg and begins taking on water. Water comes into the ship at a constant rate and some amount has already accumulated. The captain determines that 12 identical pumps could pump all of the water out in 3 hours, while five of these pumps could do it in 10 hours. To calm the passengers, the captain wants all of the water out in 2 hours. How many pumps are needed to do this? Note that water is continuing to come into the ship at a constant rate while the pumps are working.
- 6.) Fifteen students are voting to determine the best soft drink. The choices are Coca Cola, Pepsi Cola and Dr. Pepper. Each student will rank the three soft drinks first, second and third. There are three common ways to determine the winner in a three-way vote.
 - a.) Plurality. The soft drink with the most first place votes is the winner, even if this is less than half of the votes cast.
 - b.) Instant Runoff. The soft drink with the least number of first place votes is eliminated. The ballots for this eliminated soft drink are revised so that the second place soft drink is moved to first place. The soft drink with the most first place ballots after this revision is the winner.
 - c.) Point System. Two points are given for each time a soft drink is ranked first place on a ballot, one point for each time it is ranked second place on a ballot and no points for third place. The soft drink with the most points is the winner.When the ballots were cast, Dr. Pepper won when plurality was used, Pepsi Cola won when instant runoff was used and Coca Cola won when the point system was used.
Determine 15 actual ballots for which this result could occur. (There is more than one possible correct answer here.)