Order two superconvergence of the CDG finite elements on triangular and tetrahedral meshes

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Abstract

It is known that discontinuous finite element methods use many more unknowns but have the same convergence rate comparing to their continuous counterpart. In this paper, a novel conforming discontinuous Galerkin (CDG) finite element method is introduced for Poisson equation using discontinuous P_k elements on triangular and tetrahedral meshes. Our new CDG method maximizes the potential of discontinuous P_k element in order to improve the convergence rate. Superconvergence of order two for the CDG finite element solution is proved in an energy norm and in the L^2 norm. A local post-process is defined which lifts a P_k CDG solution to a discontinuous P_{k+2} solution. It is proved that the lifted P_{k+2} solution converges at the optimal order. The numerical tests confirm the theoretic findings. Numerical comparison is provided in 2D and 3D, showing the P_k CDG finite element is as good as the P_{k+2} continuous Galerkin finite element.