Formalizing the Isoperimetric Inequality in Lean 4

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Abstract

With the rapid development of artificial intelligence, mathematicians, particularly geometricians, must decide how to engage with these tools. Just as the introduction of calculators and mathematical software transformed practice, AI presents both opportunities and challenges. Until recently, automated systems offered little assistance with complex mathematical proofs, primarily due to the lack of rigorous reasoning and machine-verifiable precision. We address this gap using Lean 4, a proof assistant that enables formally verified proofs in mathematics, with a focus on differential geometry, an area that has gained renewed attention in the context of modern machine learning. Formalizing proofs in Lean 4 is non-trivial, as the structure of computer-verified proofs differs substantially from traditional handwritten ones. In this work, we present an introduction to Lean 4 and demonstrate its use by formalizing the classical isoperimetric inequality, which states that among all closed curves of a given length, the circle encloses the maximum area.