

# A Series of Rosenberg-Stolz Conjectures

Jie Xu  
New York University

Let  $X$  be a closed, oriented manifold with  $\dim X \geq 2$ . In 1994 and 2006 survey articles, Rosenberg and Stolz made the following conjectures:

*Conjecture: If  $X$  admits no metrics of positive scalar curvature, then  $M = X \times Y$  admits no complete metrics of (uniformly) positive scalar curvature, where  $Y = \mathbb{R}, \mathbb{R}^2$  or  $\mathbb{S}^1$ , respectively.*

There are two major methods in the study of scalar curvature problems on hypersurfaces: one is the method of index-theoretic obstructions; the other is geometric measure theory method initiated by Schoen-Yau minimal hypersurfaces method, and later developed by e.g. Gromov's  $\mu$ -bubble method.

In this talk, we introduce a conformal geometry and partial differential equation approach to partially answer the Rosenberg-Stolz conjectures for all dimensions. Via this approach, we also generalize the Gromov-Lawson type scalar and mean curvature comparison theorems on compact cylinders, i.e.  $Y = [0, 1]$ . The keywords are *auxiliary space* and *a new conformal invariance*. Our approach can be briefly summarized by the following diagram:

$$\begin{array}{ccc} (M, g) & \xrightarrow{\tau_1} & (M \times \mathbb{S}^1, g \oplus dt^2 = \bar{g}) \\ \iota \uparrow & & \sigma \uparrow \\ (X, \iota^* g) & \xrightarrow{\tau_2} & (W = X \times \mathbb{S}^1, \sigma^*(\bar{g})) \end{array} \quad (1)$$

With the introduction of the auxiliary space  $\mathbb{S}^1$ , we rely on the new ambient space  $M \times \mathbb{S}^1$  to analyze the notions of positivity for the (codimension 2) space  $X$ . The new conformal invariance is applied to study an elliptic partial differential equation constructed in  $W$ .

We also show how this conformal geometry approach can be applied to complex geometry. Some of my work involves collaboration with S. Rosenberg.