

Explicit Maximal Surfaces via a Weierstrass-Type Representation in Minkowski Five-Space

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Abstract. We present a Weierstrass-type construction for space-like maximal surfaces in five-dimensional Minkowski space. Using holomorphic data, the method provides a direct framework for generating conformal maximal immersions and for identifying the regular space-like region of the resulting surfaces. We construct explicit polynomial and transcendental families and illustrate how their geometry changes under projections onto different three-dimensional coordinate subspaces. We also discuss an integral-free formulation that expresses the immersion directly in terms of holomorphic functions and their derivatives, making the construction suitable for symbolic computation and visualization. These examples extend classical Weierstrass-type ideas to higher-dimensional Lorentzian geometry and provide a practical approach to the study of explicit maximal surfaces in Minkowski five-space.

Keywords: maximal surfaces; Minkowski five-space; Weierstrass-type representation; Lorentzian geometry; holomorphic null curves; integral-free construction.

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