Generalized Weierstrass–Enneper Representation Formula for Minimal Surfaces in Four-Dimensional Space

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Abstract

We present a new formulation of the Weierstrass–Enneper representation for minimal surfaces immersed in $\mathbb{R}^4$. Although the classical representation in $\mathbb{R}^3$ is based on holomorphic data and isotropic conditions in complex space, its extension to higher dimensions requires a refined geometric framework.

Our approach relies on four holomorphic functions satisfying a natural isotropy condition in $\mathbb{C}^4$, which generate conformal minimal immersions in four-dimensional Euclidean space. This formulation preserves the essential analytic structure of the classical theory while revealing additional geometric and algebraic features specific to codimension two.

We discuss structural properties of the representation and provide explicit examples illustrating its flexibility and geometric implications. The construction offers a coherent extension of the classical Weierstrass–Enneper method and provides new tools for studying minimal surfaces in higher dimensions.