

ABSTRACT. We study proper holomorphic maps of annuli in complex Euclidean spaces, that is, domains with  $U(n)$  as the automorphism group. By the Hartogs phenomenon and a result of Forstnerič, such maps are always rational and extend to proper maps of balls. We first prove that a proper map of annuli from  $n$  dimensions to  $N$  dimensions where  $N < \binom{n+1}{2}$  is always an affine embedding. This inequality is sharp as the homogeneous map of degree 2 satisfies  $N = \binom{n+1}{2}$ . Next we find a necessary and sufficient condition for a map to be homogeneous: A proper map of annuli is homogeneous if and only if its general hyperplane rank, the affine dimension of the image of a general hyperplane, is exactly  $N - 1$ . As a corollary, we obtain a classification of homogeneous proper maps of balls. A homogeneous proper ball map takes all spheres centered at the origin to spheres centered at the origin. We show that if a proper ball map has general hyperplane rank  $N - 1$  and takes one sphere centered at the origin to a sphere centered at the origin, then it is homogeneous. Another corollary of this result is a complete classification of proper maps of annuli from dimension 2 to dimension 3. Finally, we give a complete normal form of rational proper maps of annuli of degree 2.