William Meeks, 57th Texas Topology and Geometry Conference

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Title:

Recent progress in the theory of constant mean curvature surfaces

Abstract:

In this talk I will discuss joint work with Joaquin Perez, Antonio Ros, Giuseppe Tinaglia and Pablo Mira. Joint work with Perez and Ros (and Harold Rosenberg) has led to the completion of the classification of properly embedded minimal surfaces of genus 0; these examples are planes, catenoids, helicoids and Riemann minimal examples. Joint work with Tinaglia proves that compact disks of constant mean curvature 1 embedded in $\mathbb{R}^3$ have curvature estimates away from their boundaries and that there exists a universal bound on the intrinsic radius of such disks. Consequently, any complete, simply-connected embedded surface in $\mathbb{R}^3$ with non-zero constant mean curvature must be a round sphere. This work with Tinaglia also implies that complete embedded surfaces in $\mathbb{R}^3$ of positive constant mean curvature have bounded second fundamental forms if and only if they have positive injectivity radius. We also prove that a complete embedded surface in $\mathbb{R}^3$ is proper if it has finite topology or positive injectivity radius. Joint work with Perez and Ros gives the existence of removable singularity results for constant mean curvature laminations and these results lead to a better understanding of the local structure of CMC foliations of 3-manifolds near any isolated singularity. My talk ends with an outline of my recent proof with Mira, Perez and Ros of the Hopf Uniqueness Theorem in homogenous 3-manifolds. This generalization proves that two such spheres of the same mean curvature are congruent and provides a description of the associated 1-dimensional moduli spaces.