INTEGRABLE EVOLUTION OF CLOSED VORTEX FILAMENTS: FINITE-GAP SOLUTIONS AND THEIR LINEAR STABILITY

The vortex filament equation (VFE), describing the self-induced motion of a vortex filament in a perfect fluid, is a simple but important example of integrable curve dynamics, and one in which knotted curves arise as solutions of a differential equation possessing a rich geometrical structure.

The connection between the VFE and the cubic focusing nonlinear Schrödinger (NLS) equation through the well-known Hasimoto transformation allows the use of many of the tools of soliton theory to study properties of its solutions.

In this talk I will focus on three areas: The construction of closed finite-gap filaments using the classical Baker-Akhiezer eigenfunction approach developed by Krichever for the NLS equation. The use of isoperiodic deformations of spectral data for constructing large families of periodic small amplitude finite-gap solutions of increasingly high genus, and the complete description of their knot types in terms of spectral data. The relation between the linearizations of the vortex filament and NLS equations, and the role of squared eigenfunctions in determining the stability properties of VFE solutions in terms of those of the associated NLS potentials.

This talk is based on long-term collaborative work with Tom Ivey and on a more recent collaboration with Stephane Lafortune, both at the College of Charleston.