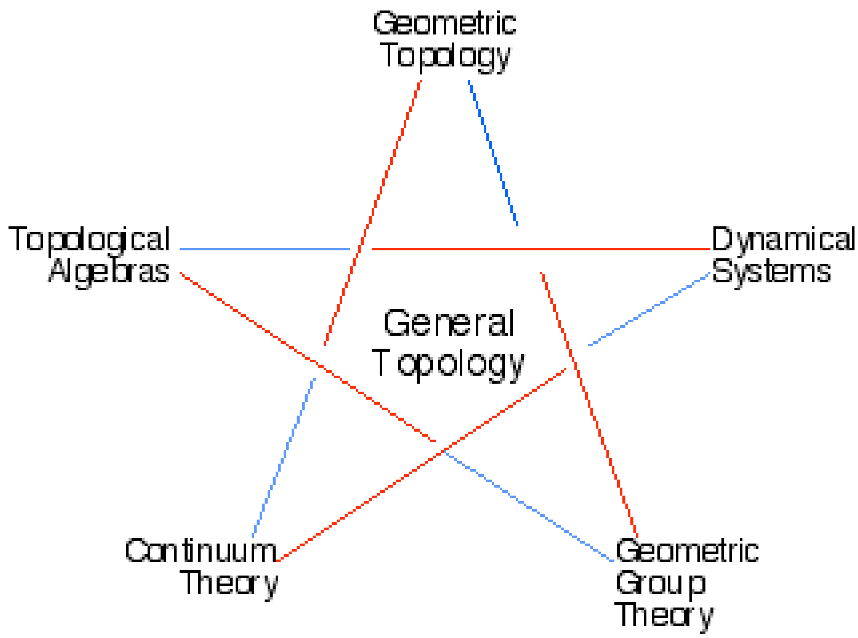
Spring Topology and Dynamical Systems Conference 2003 White Paper

The Spring Topology and Dynamical systems Conference 2003 was held at Texas Tech University March 20 - 22, 2003. At the conclusion of the conference a "white paper" discussion was held concerning major questions, challenges and future directions in the field.

This paper is the result of that discussion and of subsequent comments submitted to the conference organizers.

The conference was organized into six special sessions. For the concluding discussion brief presentations were given in the subject area of each of these sessions. This paper is also organized according to these subject areas.

However, we feel it important to emphasize that this division into subject areas is not sharp and is done largely as a matter of organizational convenience. There is much interaction between the various areas. At the conference this was readily evident by the broad attendance at plenary and invited talks and by persons presenting talks in more than one special session, as well as by the frequent switching of persons between the sessions. The organizers of the special sessions also took this overlap of interest into account when scheduling specific talks, so as to make it possible for persons from different areas to readily switch between the sessions and so as to minimize possible conflicts. This overlap of interest in the different areas was also readily evident in the interactions observed outside of the talks. The conference logo was designed to reflect these connections between various areas.



CONTINUUM THEORY

(Compiled by Wayne Lewis, incorporating comments submitted by Lex Oversteegen.)

The special session in Continuum Theory was the largest at the conference. It included a broad spectrum of both junior and senior researchers, including a significant number of students and recent Ph.D. recipients. There was especially strong representation from the very active group of researchers in Mexico. Each of these aspects should be broadly encouraged and supported.

Since the 1980's there has been increasing interaction between Continuum Theory and Dynamical Systems. At the same time there have continued to be the traditionally strong ties between Continuum Theory and General and Geometric Topology.

Areas of common interest for Continuum Theory and Dynamical Systems include the dynamics of maps of low-dimensional continua, relating to known results for the dynamics of maps of the interval. There have been several recent results on the existence of expansive homeomorphism or homeomorphisms of positive entropy and the implications of these for the existence of indecomposable continua.

Other areas of common interest include the study of minimal sets and of minimal maps of 2-manifolds (through mostly continuum theoretic techniques) and of sets which are locally the product of a 0-dimensional set and an $n-$cell (through methods of tiling spaces and symbolic dynamics).

Techniques from Continuum Theory have also proved useful in studying the dynamics of rational-like maps on locally connected or indecomposable Julia sets (using the notion of finitely Suslinean continua or finding rational maps whose Julia sets are Sierpiński curves, etc.). Moreover, the study of dynamics of maps on dendrites (inspired by the fact that dendrites occur naturally as the Julia set of complex polynomials) has added breadth and importance to the work on dendrites within Continuum Theory.

Homogeneous continua continue to be a topic of considerable interest. Every known 1-dimensional continuum is obtainable from the simple closed curve, pseudo-arc or Menger universal curve using inverse limits or continuous decompositions. It is unknown if there is any fundamentally different such continuum. While much work has been done on the classification of homogeneous plane continua, a complete solution has remained elusive, perhaps awaiting the development of new techniques. Recent investigations also concern homogeneous continua which are locally the product of an $n-$cell and a Cantor set and possible connections with higher dimensional tiling spaces and, hence, active current research in dynamical systems. Much work remains to be done on higher-dimensional homogenous continua. A number of major questions involve homogeneous embeddings of continua. It is expected that indecomposable continua will continue to play a central role in this area.

There is continued work on 2-manifolds. This includes work on the classic plane fixed point problem and the problem of extensions of isotopies of non-separating plane continua to the entire plane. In this area there are also connections with complex analysis through prime ends (and external rays), geometry (through metric versions of the Riemann map) and dynamics of rational maps of the sphere. Light open maps on $n-$manifolds are also an active area, with connections to the Hilbert-Smith conjecture and the study of minimal maps.

Central to many of these areas are questions on embeddings of continua. We still have no generally applicable method of determining when a one-dimensional continuum (or even a tree-like continuum) can be embedded in the plane. For planar continua are there methods other than prime ends or accessibility which can be used to distinguish embeddings? It is known that every non-locally connected chainable continuum (i.e. nondegenerate and not an arc) admits uncountably many distinct embeddings in the plane, but is unknown to what extent a comparable result is true for other 1-dimensional planar continua. Areas needing much further investigation are which homeomorphisms of a continuum extend to homeomorphisms of a manifold containing the continuum for a given embedding and for a given homeomorphism of a continuum which embeddings of the continuum in a given manifold admit an extension of the homeomorphism to a homeomorphism of the manifold.

Hyperspaces continue to be an active area of investigation. Among areas of interest are which classes of continua are $C-$determined and the structure of the hyperspaces $C_n(X)$ and $F_n(X)$ for various continua. There is also much continuing interest in which topological properties are Whitney properties or are Whitney reversible.

There are a number of results showing that hereditarily indecomposable continua are very abundant in the sense of category and that homeomorphisms of hereditarily indecomposable continua are also abundant in the sense of category as are Bing maps (maps between continua or manifolds where every component of every fiber is hereditarily indecomposable). It has been shown that the collection of hereditarily indecomposable subcontinua of an $n-$manifold ($n \geq 2$) forms a non-Borel, coanalytic set in the collection of all subcontinua of the $n-$manifold. There are a number of very interesting questions on how generic indecomposability is in various areas of Dynamical Systems. Here it seems that frequently indecomposability is common but not hereditary indecomposability.

DYNAMICAL SYSTEMS

The special session in Dynamical Systems included talks in a number of areas, as well as by invited speakers, as indicated in the conference program. There was close interaction with the Continuum Theory special session.

At the concluding White Paper session, each of the special session co-organizers presented comments on topics of active investigation and future directions for the area.

Unfortunately, efforts to obtain a written version of these comments, or similar comments from other researchers in the area, have not produced any result.

GENERAL TOPOLOGY

(Submitted by Alan Dow)

General Topology continues its growth primarily through its role as set-theoretic partnering with set-theory. Just in getting ready to give this presentation, I was able to sit down and write out 67 quite major problems that are still of considerable interest. This field is a major consumer of new developments and techniques from set-theory and even model theory. But it is also a major generator of subtle and fundamental questions, and solutions of foundational questions, that are exposed by basic and interesting topology questions. The field is also very productively influenced by questions arising in other areas of analysis and we have interesting things to say about Banach spaces and the related compacta. This latter context was especially in evidence at this conference.

Just looking at the basic topological properties and spaces, such as metrizability; (locally) (countably) compact; first-countability, countable tightness, sequentiality; normal and collectionwise normal; Lindelöf; $\beta\omega$, $\beta\mathbb{R}$; we find many very interesting and contemporary questions. Recent breakthroughs leave us wanting more. The tools continue to evolve and get ever subtler.

A general theme of problems that are definitely getting tougher is generated by a fill-in-the-blanks statement:

If K is compact (separable or ccc) and ???, does it follow that the space K must contain $\beta\omega$, a convergent sequence, a $G_\delta-$point, a Lindelöf subspace of cardinality ω_1, or does K map onto an uncountable product?

We are still exploring the consequences of the Proper Forcing Axiom, for example, in the context of $\beta\omega \setminus \omega$, does it contain non-trivial copies of itself, is the complement of each point non-normal, is there a Borel lifting of the measure algebra? Other natural questions about $\beta\omega \setminus \omega$ are unresolved in other models: can there be points of small character without there being $P-$points, what is the general structure of a maximal almost disjoint family, can Martin's Axiom hold in a model in which $\beta\omega \setminus \omega$ maps onto all compacta of weight continuum?

A very striking question is the Scarboro-Stone problem of whether products of sequentially compact spaces remain countably compact. It is not known if there is a ZFC example of a first-countable countably compact separable space which is not compact: a question that deserves an answer in any introductory topology class.

Several very important metrizable type questions remain. Of course there is the infamous M_3 implies M_1 problem, and the question of the status of the normal Moore space question in a model of the continuum being relatively small (e.g. $\aleph_2$). Can a countable union of open metrizable subspaces be non-metrizable? If a space is normal and has a $\sigma-$disjoint base, will it be paracompact. As I suggested, these are natural difficult problems about basic concepts.

A new theme that should be picked up following the untimely demise of Z. Balogh is to continue to explore the applications of his powerful techniques of applying elementary submodels in the construction of complicated topological spaces. Each of Shelah and Woodin have introduced fundamental new forcing models and techniques, these have started to find their way into questions in topology, but obviously there will be many more.

GEOMETRIC GROUP THEORY

Though the special session on Geometric Group Theory was smaller than the other special sessions at this conference, it was quite active. It concluded with a session of open problems.

Unfortunately, we have been unable to obtain a written version either of the questions discussed at this session or of the comments submitted by the session organizers at the concluding White Paper session of the conference.

GEOMETRIC TOPOLOGY

The special session in Geometric Topology covered knot theory, manifolds and shape theory. Comments from the area at the White Paper session concentrated primarily on knot theory.

(Submitted by Dan Silver, with contribution by Louis Kauffman)

Topology, like mathematics itself, defies its boundaries. In the 100 years since Poincaré published *Analysis situs*, topology has received stimulus from nearly every area of mathematics. Biology, chemistry and physics have contributed important ideas as well. There is now a pressing need to widen the avenues of communication between topologists and researchers in mathematics and the sciences in general. It is encouraging to note that many fine topology survey papers have appeared recently (see, for example, the extraordinary collection *History of Topology*, edited by I.M. James, published by North-Holland; other examples include *A survey of applications of surgery to knot and link theory* by J. Levine and K. Orr, and *A survey of classical knot concordance* by C. Livingston, arXiv:math.GT/0307077). Papers that provide lexicons and necessary background material for researchers crossing disciplines have also appeared (e.g., *K-theory of hyperbolic 3-manifolds* by I. Nikolaev, arXiv:math.GT/0110227).

Knot theory is a microcosm of topology. The subject has drawn from so many areas, but unlike topology in general, serious applications have emerged only in recent years. Is it possible that we are thinking too literally about knots? Applications of geometry often exploit the *relations* that geometric figures represent as well as their physical aspects. The question "What is a knot?" remains basic. Louis Kauffman offers the following thoughts on the question.

"What is a Knot and What is Knot Theory?" by Louis Kauffman

Initially, for mathematicians, knot theory was a case of the placement problem: Understand the embeddings of a space X in a space Y up to isotopy of embeddings. This uniform definition included low and high dimensional versions of knot theory, and it was understood that the special case of embeddings of circles in $\mathbb{R}^3$ had many special and fascinating properties that made it a touchstone for other investigations. The diagrammatic work of Reidemeister and Alexander was regarded as a useful technical reformulation of the classical knot theory in combinatorial terms.

Since the advent of the Jones polynomial and the state summation and partition function approaches to knot invariants, the picture of the combinatorial approach to knot theory has changed. One realizes that the combinatorial approach puts knots and their diagrams in the position of a pivot between many different mathematical and physical subjects such as Lie algebras, Hopf algebras, statistical mechanics and quantum field theory. One begins to recognize that the original intent to study the placement problem has been replaced by a use of knot theory as a nexus for questions in a collection of disciplines. In a sense, we no longer know what is a knot! Is a knot an embedding or is it a part of an interdisciplinary diagrammatic language? The answer is both, and there are new generalizations of the classical ideas waiting to be articulated. For example, many problems involve a mixture of topological and geometric moves (e.g. the theory of rigid vertex graphs and its relationship with Vassiliev invariants and with generalizations of Lie algebras). Combinatorial frameworks are just right for these sorts of models, since the general picture is a category of combinatorial objects subject to certain moves. One searches for invariants of these moves. It is clear to us that knot theory is a *seed* for a wide generalization of this *new* combinatorial topology to large areas of mathematics and applications.

Another side of the question "What is a knot?" is the matter of direct physical modeling of knots. For this one must add physicality in the form of tensile strength, flexibility, thickness and other physical parameters. Physical knot theory in this sense is now a highly active field interfacing with polymer and DNA research and with many computer studies that are giving us information about problems of dynamical differential geometry (e.g. to find the minimal energy configurations of knots an links under self-repelling forces) that are of great interest and present extraordinary analytical difficulties.

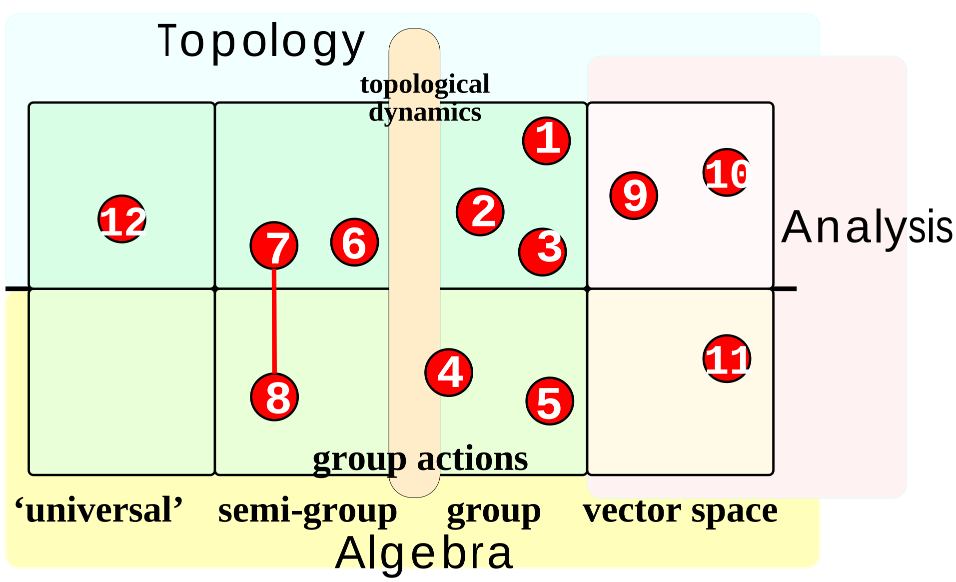
TOPOLOGICAL ALGEBRAS

(Submitted by Alexander Arhangelskii and Paul Gartside)

Introduction

Topological algebra lies at the intersection of topology and algebra. It studies objects with linked topological and algebraic structure. For example consider all rotations of the unit sphere. This is both a group (under composition), and a topological space (it is intuitively clear when two rotations are 'close').

Clearly 'Topological algebra' is a broad field. Its general principles unify mathematics and, to a great extent, are reponsible for its architecture. To bring some order to the following discussion we (1) consider algebraic structures in order of increasing strength ('universal', semi–group, group, vector space) and (2) divide results into those which focus on *topological* consequences and those which emphasize *algebraic* consequences.



We start our discussion with groups. Then we continue with semi–groups and their fascinating applications in number theory. The third topic will be vector spaces. And conclude with some brief comments on topological 'universal algebra'.

In each case we highlight just a very few recent results which we feel give an indication of where the field is right now, and point the way forward.

Groups/Topology

A central and steadily developing topic in topological algebra is that of the topological properties of topological groups — thinking of the algebraic structure as a 'boundary condition'. Here we highlight two recent and significant developments, and talk about prospects for progress in a third area. Then, despite our best attentions for brevity, we discuss some other open problems we feel are especially interseting, and briefly mention some highlights concerning free topological groups.

Productivity (1): One of the most exciting results announced at the Spring Topology Conference was of a Lindelof topological group with non–normal square by Pavlov. Earlier examples had been constructed under additional set theoretic assumptions. There are a number of similar questions currently in set theoretic limbo, for example: is there in ZFC a countably compact topological group with non–countably compact square? Pavlov's example is a subgroup of $C_p(X, \{0,1\})$ which reminds us of a very concrete question of this type: is there a space X so that $C_p(X)$ is Lindelof but $C_p(X)^2$ is not Lindelof? The example raises the possibility that numerous questions concerning productivity of topological properties in topological groups may be close to being settled. More progress in Set Theory developing combinatoric principles valid in ZFC (ZFC club principles, such as those obtained by Shelah from PCF theory, for example) might be highly useful for finding ZFC examples of topological groups with unexpected combinations of properties.

Topologizing Groups (2): Another intensively studied area has been the characterization of which groups admit a topology with specified topological properties. This is closely related to the lattice of admissible topologies (with specified topological properties) on a given group. Recently Dikranjan and Shakmatov constructed a model of set theory in which they can characterize algebraically Abelian groups admitting a countably compact group topology. Their construction of this model of ZFC by forcing is remarkable in that both topology and algebra are controlled. Previous forcing constructions have forced *either* topological *or* algebraic consequences. The Dikranjan & Shakmatov result is also notable for being a consistency *theorem* rather than a consistent counter–example.

Automatic Continuity (3): A variation on the preceding theme is that of determining when the topological structure is determined by the algebraic structure. Or if group homomorphisms are automatically continuous. For example, in a finitely generated pro–p group a subgroup of finite index is necessarily open (Serre). Thus we know exactly what the open subgroups are, and so know everything about the topology. It follows that any group homomorphism between two finitely generated pro–p groups is necessarily continuous, and hence the group of topological automorphisms coincides with the group of algebraic automorphisms. A similar result holds for Banach algebras of the form $C(K)$. Namely, it is consistent and independent that an algebra homomorphism between such Banach algebras is automatically continuous (Dales & Woodin). Automatic continuity results also hold for various automorphism groups. This may be an area ripe for methodical investigation. Evidently progress would have major impacts on pro–finite group theory, functional analysis, and logic/combinatorics. For a concrete question take that of Serre: is every subgroup of finite index necessarily open in a finitely generated pro–finite group?

Some Other Open Problems:

- The concept of completion plays a fundamental role in mathematics. In General Topology two closely related constructions of this kind are especially important: Dieudonne completion and Hewitt-Nachbin completion. A natural basic question is: given a topological group G, when the algebraic structure of G can be continuously extended to the Dieudonné completion μG of the space G? Arhangel'skii has shown that such an extension is not always possible. It remains unknown if the extension is possible for arbitrary subgroup of the product of an uncountable family of separable metrizable groups. This is an outstanding open problem.

- Another vexing open problem concerning convergence in topological groups is whether there exists in ZFC an example of a countable Fréchet-Urysohn topological group G which is not metrizable.
- For over 30 years the question whether there exists in ZFC an extremally disconnected non-discrete topological group has remained open.

Free Topological Groups: Every Tychonoff space X naturally generates the free topological group $F(X)$ of the space X. The algebraic structure of $F(X)$ is standard, but the topology of $F(X)$ has most non-trivial, even enigmatic, behaviour. In recent years, in works of Tkachenko, Uspenskij, Sipacheva, Yamada, the structure of this topology has become much better understood. In particular, Sipacheva established that the free Abelian topological group of a metrizable space is always stratifiable, and Tkachenko proved that the free topological group of arbitrary Dieudonne complete space is complete with respect to its natural uniform structure. The proofs of these facts are difficult and delicate.

Groups/Algebra

Lie groups have their own rich theory, and are not considered here. Instead the focus here is on 'naturally occurring' topological groups which are definitely not Lie. The many examples from Banach space theory will be considered later. For now let us look at automorphism groups — the groups of symmetries of some structure. According to the Erlangen program, the symmetries of an object are more important (more revealing) than the object itself. Most automorphism groups of infinite structures carry with them a natural (non–trivial) group topology. A classic example is that of Galois groups which carry a compact group topology (Krull topology). But most reasonably small structures have a natural Polish (separable, completely metrizable) topology on their automorphism group.

Automorphism Groups (4): The study of automorphism groups of first order structures is a highly fruitful meeting point between topological algebra, algebra/combinatorics and logic. One of the mathematical highlights of 2002 was the announcement by Knight of his solution of the *Vaught Conjecture*. In terms of automorphism groups this means there is a first order theory M whose automorphism group acting on the space of countable models has precisely $\aleph_1$ orbits. This implies the theory has just $\aleph_1$ countable models. This is extraordinary... Knight's (who is a topologist) construction (140 pages long) is a testament to the power of the techniques and strategies developed by topologists.

Another recent result of considerable interest is due to Shelah: he showed that no non–discrete automorphism group can be isomorphic to a free group. This should open up the way to many more results about the algebraic structure of automorphism groups, and perhaps Polish groups in general.

More input by topologists into the study of (infinite) Galois groups (5) is needed. Uncountable extensions lead to non–metrizable compact groups — an area exhaustively investigated by many on the more topological side of topological algebra.

Group Actions

Automorphism groups clearly come with a natural action, and the study of Polish groups acting on Polish spaces is proceding briskly led by Kechris et al. Gartside spoke at symposium about the action of an automorphism group on the space of substructures.

The topic of continuous actions on topological groups on compacta is also an area of interaction between topological algebra and topological dynamics. It has close and obvious relationship to the study of homogeneous compacta – these are compacta on which the group of all autohomeomorphisms acts transitively. The structure of homogeneous compacta remains practically unknown and it might be expected that the methods of topological algebra can help to find a breakthrough. It is even unknown whether every infinite homogeneous compact Hausdorff space contains a non-trivial convergent sequence (Walter Rudin's question, almost 50 years old). The principal achievement in this difficult domain was Uspenskij's result that if a subgroup of the product of arbitrary family of separable metrizable groups acts continuously and transitively on a compactum X, then X is a Dugundji compactum.

Semi–Groups/Topology

The algebraic structure appears to place little restriction on the topology of a topological semi–group. Consequently most attention is paid to (countably) compact semi–groups. In applications the semi–group operations may only be separately continuous, and this is addressed by the theory.

Separate and Joint Continuity: The question of when separately continuous algebraic operations are jointly continuous is a crucial problem in semi–group theory, group theory and function space theory. The techniques of set–theoretic topology have opened new doors here. In particular, the notion of fragmentability, which is a far reaching generalization of the notion of scattered space, has lead I. Namioka, R. Pol. R. Haydon, J. Jayne, A. Bouziad, P. Kenderov and others to remarkable new results on separate and joint continuity and their applications. A survey of this topic was presented at the conference by Z. Piotrovskij.

Countably compact semi–groups (6): The key question is that of Wallace: is a countably compact cancellative semi–group a group? A few years ago Robbie and Svetlichny showed that this is true under CH. But is there a ZFC example or a consistency theorem?

Compact semi–groups (7): Compact right topological semi–groups have a rich algebraic theory. Witness the book *Algebra in the Stone–Cech Compactification* by Hindman & Strauss. The existence of idempotents, for example, makes this topic quite different from the theory of compact groups. There are fascinating connections with set theoretic topology (Stone–Cech compactification, ultrafilters), and, as we discuss next, number theory.

Semi–Groups/Algebra (8)

Given a discrete semigroup S, its Stone-Cech compactification βS can be made into a right topological semigroup in a natural way. As such it has all of the structure of any compact right topological semigroup. This means that the tools of topological algebra can be applied to the study of such basic objects as $\mathbb{N}$ under addition. In particular it has a smallest two sided ideal which is the union of all minimal right ideals and also is the union of all minimal left ideals. The intersection of any minimal left ideal and any minimal right ideal is a group. These groups are commonly quite large. For example in $\beta\mathbb{N}$, these groups all contain copies of the free group on $2^{\mathfrak{c}}$ generators. Particularly powerful are the 'minimal idempotents', that is idempotents in the smallest ideal. Their members are the 'central' sets, and enjoy elaborate combinatorial properties.

This structure has turned out to be a remarkably powerful tool for obtaining combinatorial results about the original semigroup. The result are extremely efficient proofs of such classical results as that of van der Waerden: in any partition of $\mathbb{N}$ into finitely many pieces, at least one piece contains arithmetic progressions with unbounded lengths. And many new results, some of which do not have 'classical' proofs.

Vector Spaces

The objects of attention are: the function spaces $C_p(X)$ and $C_k(X)$, Banach spaces in the weak topology, and the Banach space $C(K)$. Thus this topic is as much analysis as topological algebra. A number of exciting advances were announced at the Spring Topology Conference, and we can't resist talking about them here. The basic thrust of these results is that analytic topology, especially the study of compacta, has a lot to offer to functional analysis.

Function Spaces (9): The function spaces $C_p(X)$ and $C_k(X)$ are topological locally convex vector spaces, and algebras. They have been studied intensively for their self–evident interest (after the reals, $\mathbb{R}$, surely the central object of study of analysis is the space of all continuous real valued functions, $C(X)$), and for their connection with Banach spaces in the weak topology.

There is now a deep theory around $C_p(X)$, and a somewhat less developed theory for $C_k(X)$. An amazing new result was exposed by Okunev in his plenary talk: a necessary condition for $C_p(X)$ and $C_p(Y)$ to be homeomorphic, where X and Y are two compacta, is that the tightness of X and Y is the same. Despite the many, many advances, some fundamental questions tantalize: if $C_p(X)$ and $C_p(Y)$ are homeomorphic then do the dimensions of X and Y coincide? More concretely: which of $C_p(\text{Cantor set})$, $C_p([0,1])$ and $C_p([0,1]^2)$ are homeomorphic? In a different direction, as explained in a talk by Gruenhage, spaces of the form $C_k(X)$, for X separable metric, may hold the key to answering the famous M_1–M_3 problem of general topology.

Eberlein Compacta, Isomorphism of $C(K)$ (10): A space homeomorphic to a compact subspace of a Banach space (resp. Hilbert space) in its weak topology is called an 'Eberlein' compactum (resp. 'strong Eberlein' compactum). They play a key role in functional analysis, and C_p–theory. In his plenary address, Marciszewski announced, jointly with Bell, that Eberlein compacts of scattered height $\leq \omega + 1$ are uniformly Eberlein compact. He also announced a characterization of compacta K so that $C(K)$ is isomorphic to a classical sequence space $c_0(\Gamma)$. An interesting corollary is an example of an Eberlein compactum L so that $C(L)$ and $c_0(\omega_\omega)$ are bilipschitz isomorphic, but $C(L)$ is not isomorphic to any $c_0(\Gamma)$. Koszmider announced the consistent construction of a compactum K so that $C(K)$ is not isomorphic to $C(K')$ for any zero–dimensional K'.

Structure of $C(K)$ (11): Koszmider's plenary address tackled some fundamental problems in the structure of Banach spaces in general, and $C(K)$ in particular. In ZFC there is a compactum K so that $C(K)$ has the quite bizarre property that it is not isomorphic to any of its proper closed linear subspaces, or to any of its proper quotients. Assuming the Continuum Hypothesis there is a compactum K so that every bounded linear operator on $C(K)$ is of the form $gI + S$, where g is a continuous function on K and S is weakly compact. Probably the key question here is whether there is a Banach space, and more particularly a $C(K)$, so that every bounded linear operator has the form $cI + C$ where c is a constant and C is compact.

'Universal Algebra'

Beyond the 'classical' algebraic structures (groups, rings etc) there are a number of rather weak algebaric systems which seem to have been overlooked by those of us in topological algebra. It seems even such basic questions as how to define what is a 'topological associative scheme' are not fully answered.

Some progress has been made with 'topological universal algebra'. Particularly in the context of studying precisely what algebraic structure is needed for such classical results as 'compact topological groups are dyadic' and 'first countable topological groups are metrizable'. It could be very interesting to *prove* that certain algebraic systems are minimal for certain theorems (rather than constantly finding incremental improvements).

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