Structures on free resolutions

16th Red Raider Mini-Symposium Lubbock, Texas October 26-28, 2017

Speakers

Adam Boocher, University of Utah Daniel Erman, University of Wisconsin-Madison Andrew Kustin, University of South Carolina Jason McCullough, Iowa State University Claudia Miller, Syracuse University Alexandra Seceleanu, University of Nebraska-Lincoln Liana Şega, University of Missouri-Kansas City Mark E. Walker, University of Nebraska-Lincoln Jerzy Weyman, University of Connecticut

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SCHEDULE AND ABSTRACTS FOR "STRUCTURES ON FREE RESOLUTIONS"

OCTOBER 26-28, 2017

THURSDAY, OCTOBER 26

4:30pm Colloquium reception

5:30pm Speaker: Mark Walker

Title: The total and toral rank conjectures

Abstract: Assume X is a compact CW complex that admits a fixed-point free action by a d-dimensional torus T. For example, X could be T acting on itself in the canonical way. The Toral Rank Conjecture, due to Halperin, predicts that the sum of the ranks of the rational homology groups of X must be at least 2^d . There is a variant of this conjecture, due to Carlsson, concerning spaces that admit a fixed-point free action by an elementary abelian p-group.

Now assume R is a local ring of dimension d and F is a complex of free R-modules whose homology modules are non-zero and of finite length. It has been conjectured that the total rank of F (i.e., the sum of the ranks of the free modules occurring in F) must be at least 2^d . When F is the minimal resolution of a non-zero module of finite length, this conjecture is a weak form of the well-known Buchsbaum-Eisenbud-Horrocks Conjecture.

In this talk I will discuss the relationship between these conjectures and recent progress toward settling them.

FRIDAY, OCTOBER 27

9am Speaker: Jason McCullough

Title: Bounds on Invariants of Prime Ideals

Abstract: Given a homogeneous, nondegenerate prime ideal P in a polynomial ring S, it was conjectured in the 80's by Eisenbud and Goto that the Castelnuovo-Mumford regularity of P was bounded by the multiplicity or degree of the associated projective variety (minus a correction term involving the codimension). Recently with I. Peeva, we constructed counterexamples that show that no polynomial bound on the regularity of homogeneous, nondegenerate prime ideals purely in terms of multiplicity exists. In my talk I will review the construction and explain more recent work, joint with I. Peeva, G. Cavaglia and M. Varbaro, which shows there is a bound (obviously not polynomial) on the regularity of primes in terms of multiplicity. I will also describe some other extensions to the previous work and some open questions.

10am Speaker: Adam Boocher

Title: Inequalities on Betti Numbers

Abstract: In this talk I'll discuss two inequalities concerning the Betti numbers of graded algebras. One, the Buchsbaum-Eisenbud, Horrocks rank conjecture says roughly that the Koszul complex is the "smallest" possible resolution. The second, a conjecture of Conca for Koszul rings, is motivated by the Taylor resolution for monomial ideals. I'll discuss recent work on these topics and how the algebra structure of Koszul homology has given us hints about the structure of these algebras. 11am Speaker: Seth Lindokken

Title: Matrix factorizations and Betti numbers of finite length modules

Abstract: The BEH Conjecture (from Buchsbaum, Eisenbud, and Horrocks) predicts that if (R, \mathfrak{m}, k) is a regular local ring of dimension d and if $M \neq 0$ is a finite length R-module, then there is an inequality $\beta_i^R(M) \ge \binom{d}{i}$ for all i. The validity of the BEH conjecture would imply that, in the setting above, there is an inequality $\sum_{i=0}^{d} \beta_i^R(M) \ge 2^d$. The prediction that this inequality holds is known as the Total Rank Conjecture (TRC), and was recently verified by Walker when char $k \neq 2$. In this talk we will explore a version of the TRC for matrix factorizations. By extending arguments of Charalambous, we verify the conjecture in certain multigraded settings. However, using a construction from Iyengar-Walker, we show that the conjecture does not hold in the local case.

11:30am Break

12pm Workshop led by Mark Walker

1pm Lunch

2:30pm Speaker: Matthew Mastroeni

Title: Koszul almost complete intersections

Abstract: Let R = S/I be a quotient of a standard graded polynomial ring S by an ideal I generated by quadrics. If R is Koszul, a question of Avramov, Conca, and Iyengar asks whether the Betti numbers of R over S can be bounded above by binomial coefficients on the minimal number of generators of I. Motivated by previous results for Koszul algebras defined by three quadrics, we give a complete classification of the structure of Koszul almost complete intersections and, in the process, give an affirmative answer to the above question for all such rings.

3pm Speaker: Jerzy Weyman

Title: Finite free resolutions and root systems

Abstract: In this talk I will discuss the structure of free resolutions of length 3 over Noetherian rings. Associate to a triple of ranks (r_3, r_2, r_1) in our free complex a triple $(p, q, r) = (r_3 + 1, r_2 - 1, r_1 + 1)$. Associate to (p,q,r) the graph $T_{p,q,r}$ (three arms of lengths p - 1, q - 1, r - 1 attached to the central vertex). The main result is the explicit construction of a generic ring R_{gen} for resolutions of the format with the differentials of ranks r_1, r_2, r_3 . This ring deforms to a ring that carries an action of a Kac-Moody Lie algebra associated to the graph $T_{p,q,r}$. In particular the ring R_{gen} is Noetherian if and only if $T_{p,q,r}$ is a Dynkin graph. I will discuss the structure of the ring R_{gen} and possible consequences for the structure of perfect ideals of codimension 3.

4pm Break

4:30pm Speaker: Juliette Bruce

Title: Asymptotic Syzygies in the Semi-Ample Setting

Abstract: In recent years numerous conjectures have been made describing the asymptotic Betti numbers of a projective variety $X \subset \mathbb{P}^r$ as the embedding line bundle becomes more ample. I will discuss recent work attempting to generalize these conjectures to the case when the embedding line bundle becomes more semi-ample. (Recall a line bundle is semi-ample if a sufficiently large multiple is base point free.) In particular, I will discuss how the monomial methods of Ein, Erman, and Lazarsfeld for proving non-vanishing results on \mathbb{P}^n can be extended to prove non-vanishing results for products of projective space."

5pm Speaker: Daniel Erman

Title: GL-noetherianity and Stillman's Conjecture

Abstract: I'll talk about a connection between GL-noetherianity results like those of Sam-Snowden and Stillman's Conjecture. This is joint work with Sam and Snowden.

Saturday, October 28

9am Speaker: Liana Şega

Title: Golod homomorphisms: constructions and applications

Abstract: The existence of Golod homomorphisms between certain commutative local rings is useful towards understanding homological structures. For example, the existence of a Golod homomorphism from a complete intersection onto a local ring R implies that all finitely generated R-modules have rational Poincare series, sharing a common denominator. We will discuss methods of constructing such homomorphisms, and consequences.

10am Speaker: Alexandra Seceleanu

Title: Resolutions for powers of ideals and consequences on symbolic powers

Abstract: A key insight towards understanding the homological behavior of the powers of a homogeneous ideal I is that graded free resolutions of the powers of I are encoded in the bigraded resolution of the Rees algebra of I. Under certain assumptions on I, one can take this technique further to derive explicit comparison maps between the resolutions of the powers of I. I will illustrate how such comparison maps can be used for establishing (non)containments between various symbolic and ordinary powers of the ideal I.

11am Speaker: Robert M. Walker

Title: Uniform Symbolic Topologies in Normal Toric Rings

Abstract: A Noetherian ring R has the uniform symbolic topology property (USTP) if there's an integer D := D(R) > 0 such that the symbolic power $P^{(DN)} \subseteq P^N$ for all prime ideals P in R and all integers N > 0. For instance, all excellent finite-dimensional regular rings have USTP, and a large class of isolated singularities also have USTP (Ein-Lazarsfeld-Smith, Hochster-Huneke, Huneke-Katz-Validashti, Ma-Schwede). A toric ring is a domain of finite type over a field, generated by Laurent monomials. In this talk, we present a formula for the multiplier D(R) such that any normal toric ring R has USTP on the set of monomial primes: this is one of the conditional USTP results my dissertation affords for rings whose singular locus may have positive dimension.

11:30am Break

12am Workshop led by Daniel Erman

1pm Lunch

2:30pm Speaker: Zheng Yang

Title: Weak complete intersections

Abstract: In our study of the Poincare series of a certain fiber product of commutative local rings, we are led to investigate a family of ideals, with the property that every differential in their minimal free resolutions can be represented by a matrix whose entries are in the ideal itself. These ideals share similar formulas with those for the maximal ideals and bring us possible new examples of Golod rings.

3pm Speaker: Claudia Miller

Title: Generalized Koszul complexes and applications to resolutions

Abstract: We will discuss the history of some ubiquitous complexes, that usually go by the name of generalized Koszul complexes, and how they have re-appeared in some recent work. First we recall original constructions by Buchsbaum and Eisenbud, as well as Lebelt, Bruns, and Vetter, all building on work of Northcott. Then we also recall the more general double complexes used by the first group to obtain minimal resolutions of powers of ideals of variables.

Then we will concentrate on applications: one by El-Khoury and Kustin to the construction of resolutions of Artin algebras, extended further in our joint work with Rahmati, and, perhaps if time permits, another in older work of Lebelt and Vetter, as well as more recent work of Graf and joint work with Vassiladou on the study of modules of differentials over complete intersections. 4pm Break

4:30pm Speaker: Rebecca R.G.

Title: Betti numbers of Frobenius powers of ideals in characteristic p > 0 (joint with Claudia Miller and Hamid Rahmati)

Abstract: Abstract: Let $R = k[x_1, \ldots, x_d]/(f)$ where k is a field of characteristic p > 0, and f a nonzero element of R. In the case where d = 3 and $f = x_1^n + x_2^n + x_3^n$, work of Kustin, Rahmati, and Vraciu indicates that the syzygies of p^e th bracket powers $I^{[p^e]}$ of $I = (x_1^N, x_2^N, x_3^N)$ cycle through a finite number of modules as e increases. We examine the behavior of the betti numbers of $I^{[p^e]}$ when d = 3 but f is chosen generically, using the method of finding resolutions via inverse systems as developed by El Khoury-Kustin and Miller-Rahmati.

5pm Speaker: Andy Kustin

Title: Use a Macaulay inverse system to detect an embedded deformation

Abstract: Let k be a field, P be a standard-graded polynomial ring in four variables over k, and A = P/I be an Artinian Gorenstein k-algebra of embedding dimension four which is defined by six homogeneous forms and has socle degree three. (There is a (fairly minimal) restriction on the characteristic of k.) We use the homogeneous Macaulay inverse system that corresponds to I to prove that A has an embedded deformation. In other words, we prove that there is an ideal J in P and an element f in P with f regular on P/J and I = (J, f). (It quickly follows that f is a quadratic form in P and that J is a 5-generated grade three Gorenstein ideal generated by the maximal order Pfaffians of a 5×5 alternating matrix of linear forms.)

The previous best results describing the structure of six-generated grade four Gorenstein ideals assume that P/I is a generic complete intersection. We treat the case when P/I is a graded Artinian algebra over a field at the expense of assuming that the socle degree of A is three.

This talk is a preliminary report on recent joint work with Sabine El Khoury.