Complex Analysis, Potential Theory, Special Functions & Applications

The 14^{th} Annual Red Raider Mini-Symposium
Celebrating the work of Roger W. Barnard
November 6-9, 2014

Department of Mathematics and Statistics
Texas Tech University

The Red Raider Mini-Symposium Series

This series was initiated in 2001 by *Prof. Frits Ruymgaart*, a Paul Whitfield Horn Professor, who generously used his professorship's endowment to partially fund the mini-symposium's activities.

Invited lecturers at the mini-symposium include distinguished scholars who made a great impacts in the field, as well as early career mathematicians and promising young scientists. The lectures expose the audience to current research problems, as well as their solutions and applications. The topics covered by the Red Raider mini-symposia span over broad areas of pure and applied mathematics and reflect the strengths of the scientific groups in the Department of Mathematics and Statistics at Texas Tech University. In chronological order, the previous twelve symposia reflect a large diversity of research interests:

2001 *Control Theory in the Twenty-First Century*
2002 *Contemporary Algebra and Algebraic Geometry*
2003 *Mathematical and Computational Modeling of Biological Systems*
2004 *Invariant Theory in Perspective*
2005 *Geometry, Statistics and Image Analysis*
2006 *Mathematical Modeling of Novel Material and Devices*
2007 *Conformal Mapping, Circle Packing and Applications*
2008 *The Topology and Geometry of Physics*
2009 *Non-linear Analysis, PDEs and Applications*
2010 *Mathematical Modeling in Population Biology and Epidemiology*
2011 *High Level Mathematical Software for PDE's FEniCS'11*
2012 *Computational and Theoretical Challenges in Interdisciplinary Predictive Modeling Over Random Fields*
2013 *Aspects of Fluid Dynamics*

Complex Analysis, Potential Theory, Special Functions & Applications

The Red Raider Symposium is now in its 14^{th} year of bringing together world class researchers and young investigators. This year's theme is Complex Analysis, Potential Theory, Special Functions and Applications. The 5 plenary speakers will give one-hour talks spread across three days, with half-hour talks from speakers from across the United States, Canada, and Europe.

Titles and Abstracts

Catherine Bénéteau, *University of South Florida*

Women in mathematics

ABSTRACT. In this talk, I will discuss issues for women, men, and families in the mathematical community, drawing on my own personal experiences, starting with my undergraduate education at McGill University in Canada, through my experiences as a graduate student in the US, then as a professional mathematician and mathematician family member. This talk is geared to a wide audience, including women, men, undergraduate and graduate students, and faculty members.

Michael Dorff, *Brigham Young University*

Movies and math - the past, present, and future

ABSTRACT. What's your favorite recent movie? *Frozen? The Avengers? Avatar? Transformers?* What do these and all the highest earning Hollywood movies since 2000 have in common? Mathematics! You probably didn't think about it while watching these movies, but math was used to help make them. In this presentation, we will discuss how math is being used to create better and more realistic movies. Along the way we will discuss some specific movies and the mathematics behind them. We will include examples from Disney's 2013 movie *Frozen* (how to use math to create realistic looking snow) to Pixar's 2004 movie *The Incredibles* (how to use math to make an animated character move faster). Come and join us and get a better appreciation of mathematics and movies.

CATHERINE BÉNÉTEAU, *University of South Florida*

Cyclicity of polynomials of two variables in the Dirichlet Spaces

ABSTRACT. In this talk, I will give a characterization of polynomials in two complex variables that are cyclic with respect to the coordinate shifts acting on Dirichlet-type spaces in the bidisk, which include the Hardy space and the Dirichlet space of the bidisk. The cyclicity of a polynomial depends on both the size and nature of the zero set of the polynomial on the distinguished boundary. The techniques in the proof come from real analytic function theory, determinantal representations for stable polynomials, and harmonic analysis on curves. This talk is based on a paper that is joint with Greg Knese, Lukasz Kosinski, Conni Liaw, Daniel Seco, and Alan Sola.

MICHAEL DORFF, *Brigham Young University*

Planar harmonic mappings

ABSTRACT. Complex-valued harmonic mappings can be regarded as generalizations of analytic functions. We are interested in harmonic mappings that are univalent. In this talk we will provide background material, discuss some properties related to dilatations and convolutions, and present some open problems in the field.

Dmitry Khavinson, *University of South Florida*

"Fingerprinting" the lemniscates

ABSTRACT. The emerging field of vision and pattern recognition often focuses on the study of two dimensional "shapes", i.e. simple, closed smooth curves. A common approach to describing shapes consists in defining a natural embedding of the space of curves into a metric space and studying the mathematical structure of the latter. Another idea that has been pioneered by Kirillov and developed recently among others by Mumford and Sharon consists of representing each shape by its "fingerprint", a difeomorphism of the unit circle. Kirillov's theorem states that the correspondence between shapes and fingerprints is a bijection modulo conformal automorphisms of the disk. In this we shall discuss the joint work with P. Ebenfelt and Harold S. Shapiro (2011), and more current project with A. Vasiliev, outlining an alternative interpretation of the problem of shapes and Kirillov's theorem based on finding a set of natural and simple fingerprints that is dense in the space of all diffeomorphisms of the unit circle. This approach is inspired by the celebrated theorem of Hilbert regarding approximation of smooth curves by lemniscates. We shall sketch proofs of the main results and discuss some interesting function-theoretic ramifications and open-ended questions regarding possibilities of numerical applications of this idea.

David Minda, *University of Cincinnati*

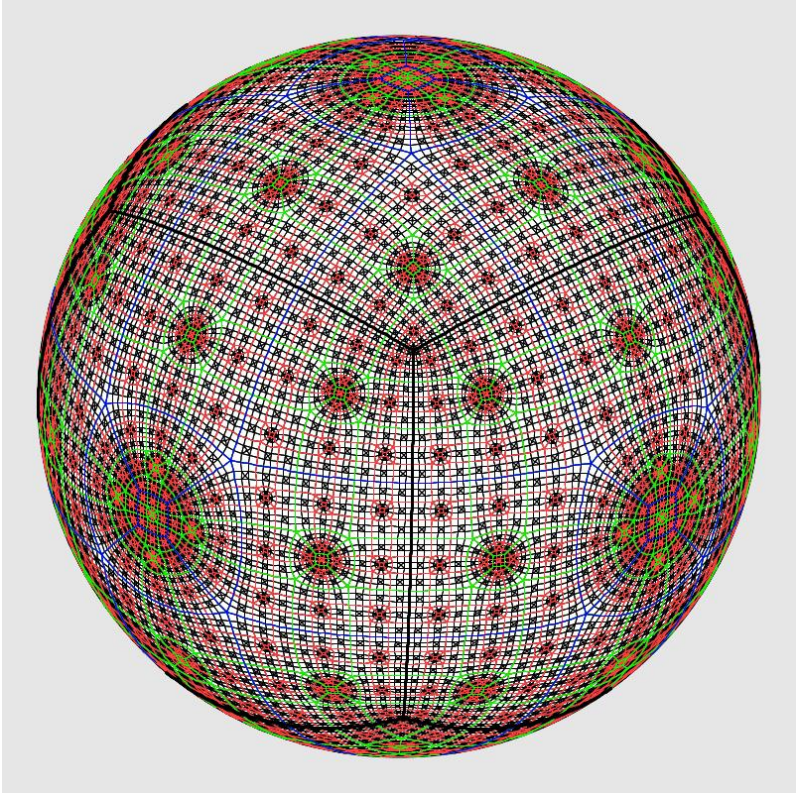
Is it geometric function theory or conformal geometry?

ABSTRACT. This largely expository talk will discuss a notion of "equivalence" between geometric function theory and two-dimensional conformal geometry. The basic result is due to Liouville in 1853 who gave an equivalence between constant curvature conformal metrics and locally injective holomorphic (meromorphic) functions. Liouville's proof fails to meet current standards for rigor. Rigorous modern proofs of Liouville's Theorem are available, but use ideas from outside function theory. I will indicate a proof that solely relies on function theoretic ideas.

KEN STEPHENSON, *University of Tennessee, Knoxville*

Conformal Tiling: foundations, theory, and practice

ABSTRACT. (Joint work with Phil Bowers) Conformal tilings represent a new chapter in the theory of aperiodic heierarchical tilings. The most famous and fascinating traditional example is the Penrose tiling formed by so-called 'kites' and 'darts'. The new theory moves away tiles with individually rigid euclidean shapes to tiles that are conformally regular and get their rigidity from the global pattern — a rigidity that those in function theory can appreciate. One sees much of the richness of the traditional theory, but also new connections to Cannon, Floyd, and Parry's work with subdivision rules, to Grothendieck dessin d'enfant, and to theories of emergent structure in random triangulations and quadrangulations. This talk will be rich with images. I will discuss the foundations, but will concentrate on phenomena observed in examples, including the conformal version of the Penrose tiling. The attached image shows the conformal tiling of the "Snow Sphere", a 3D analogue of von Koch's snowflake.



NADIA ASKARIPOUR, *University of Cincinnati*

Approximation by meromorphic automorphic forms

ABSTRACT. Let Δ be the unit disk, and $U \subset \Delta$. Also let Γ be a subgroup of SU(1, 1), acting properly discontinuously on U. A function $\phi : U \to \Delta$ is called an automorphic form of weight k with respect to Γ if $\phi(\gamma(z))\gamma'(z)^k = \phi(z)$, for any $\gamma \in \Gamma$. Automorphic forms have several applications for example in quantization and number theory. Also automorphic forms can be seen in different ways, for example in the form of k-differentials on Riemann surfaces. The case $k = 2$, (i.e. quadratic differentials) is of special importance because of the connection of quadratic differentials with Teichmuller theory. In this talk I will concentrate on some analytic aspects of automorphic forms. I will explain about approximation by meromorphic automorphic forms which is a work in progress.

ATUL DIXIT, *Tulane University*

Error functions, Mordell integrals and integral analogue of partial theta function

ABSTRACT. A new transformation involving the error function $\mathrm{erf}(z)$, the imaginary error function $\mathrm{erfi}(z)$, and an integral analogue of a partial theta function is given along with its character analogues. A complementary error function transformation is also obtained which, when combined with the first, explains a transformation in Ramanujan's Lost Notebook termed by Berndt and Xu as the one for an integral analogue of theta function. These transformations are used to obtain a variety of exact and approximate evaluations of many non-elementary integrals involving hypergeometric functions. Several asymptotic expansions, including the one for a non-elementary integral involving a product of the Riemann Ξ-function of two different arguments, are obtained which generalize known results due to Berndt and Evans, and Oloa. This is joint work with Arindam Roy and Alexandru Zaharescu.

Richard Fournier, *CRM and DMS, Université de Montréal, Canada*

A radius problem for a class of functions of bounded turning

ABSTRACT. It is known that the class of normalized functions f analytic in the unit disc $D := z||z| < 1$ and satisfying there $|f'(z) - 1| < 1$ does contain some univalent but not starlike functions. In this talk, I shall discuss the value of the radius of starlikeness of this class.

Parisa Hariri, *University of Turku, Finland*

Some remarks on the visual angle metric

ABSTRACT. We compare the geometries defined by the triangular ratio metric and the visual angle metric in several simple domains of $\mathbb{R}^n$. Moreover, we study distortion properties of quasiconformal mappings with respect to these metrics. The talk is based on the preprint P. Hariri, M.Vuorinen, G. Wang: Some remarks on the visual angle metric.

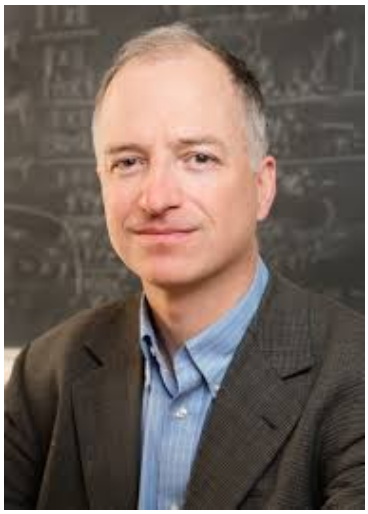
David Herron, *University of Cincinnati*

QuasiHyperbolic type metrics and universal convexity

ABSTRACT. Let $\mathcal{M}$ be a family of conformal metrics $\rho\,ds$ defined on some collection $\mathcal{D}$ of domains Ω in the n-sphere S^n. Let Ω_ρ denote the conformal deformation of Ω induced by the metric $\rho\,ds$. A non-empty open set $U \subset \mathsf{S}^n$ is *universally convex* with respect to $\mathcal{M}$ provided it has the property that for each $\Omega \in \mathcal{D}$, if $U \subset \Omega$, then U is geodesically convex in Ω_ρ. We characterize the universally convex objects for several well-known classes of conformal metrics.

Richard S. Laugesen, *University of Illinois, Urbana-Champaign*

Steklov spectral bounds through quasiconformal mappings



ABSTRACT. We show the disk maximizes functionals of the Steklov eigenvalues, among simply connected plane domains normalized by perimeter and a kind of conformal moment. The results cover the first eigenvalue, spectral zeta function, partition function, and more. Interestingly, the underlying method employs quasiconformal mappings, with conformal maps as just one special case. [Joint with A. Girouard and B. A. Siudeja.]

YUK-J LEUNG, *University of Delaware*

On the eigenvalues of a non-selfadjoint problem

ABSTRACT. An inverse scattering problem for the unit sphere in a spherically stratified medium is to determine the refractive index function $n(r)$ based on the zeros of the entire function $d(k) := y(1)\cos(k) - y'(1)\sin(k)/k$. Here $y(r)$ is the solution to a second order differential equation on $[0,1]$, $y''(r) + k^2 n(r) y(r) = 0$; with $y(0) = 0, y'(0) = 1$. The zeros of $d(k)$ as a function of k are called interior transmission eigenvalues. In the past, numerical algorithms have been set up to reconstruct the positive function $n(r)$ based on the assumption that the eigenvalues are real. Using a result of Levinson on zeros of entire functions of exponential type, we show that actually there are infinitely many complex zeros for a general index $n(r)$. If special restrictions are placed on $n(r)$, then only a finite number of complex zeros can exist. This case follows from a result of Duffin and Schaeffer.

ERIK LUNDBERG, *Florida Atlantic University*

Sheil-Small's mapping problem for polygons

ABSTRACT. The problem of mapping the interior of a Jordan polygon univalently by the Poisson integral of a step function was posed by Terry Sheil-Small (1989). In this talk, I will describe a complete solution that relies on an unexpected ingredient: the "ear clipping" algorithm from computational geometry. This is joint work with Daoud Bshouty and Allen Weitsman.

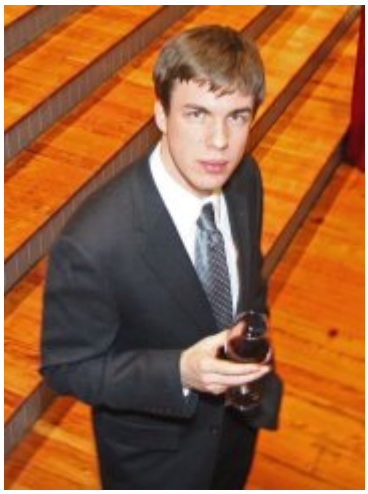
AIMO HINKKANEN, *University of Illinois, Urbana-Champaign*

Growth of the fourth Painlevé transcendents

ABSTRACT. All solutions of Painlevé's fourth equation are meromorphic in the complex plane. It has been known that the order of any transcendental solution is at least 2 and at most 4. In joint work with Ilpo Laine and Boonyong Sriponpaew, we have proved that the order of such a solution is either 2 or 4. We provide criteria for determining which case occurs. The proof is based on estimates on the behavior of the solutions in terms of an auxiliary function.

ANTON LUKYANENKO, *University of Michigan*

Cantor functions and continued fractions on the Heisenberg group



ABSTRACT. The Mobius transformations of the Heisenberg group include integer translations, dilations, and the Koranyi inversion. Translations and dilations provide an intrinsic notion of base-b expansion, and in particular Cantor functions. Translations and the inversion provide a new generalization of continued fractions. I will describe the two constructions and some open questions about them.

ERIC M. MURPHY, *U.S. Air Force*

Random Boolean network analysis as a method for illuminating complex international relations

ABSTRACT. To the extent that quantitative analyses of international conflict have been rigorously conducted and offered, they remain unsatisfying in their capacity to predict behavior. However, a set of techniques seeing increasing application in this regard is network analysis, a field that offers international relations scholars 1) a collection of theories describing relationships between the structural characteristics of networks and their behaviors and 2) a collection of analytical tools for investigating these structural properties. It is toward advancing the study of networks in international relations that this effort aims, and with this in mind borrows a tool that has proven heuristically valuable in the understanding of complex relationships in the social and physical sciences-random Boolean networks (RBNs). Not finding RBNs applied to international relations anywhere in the literature, we offer the following in the expectation that it will illuminate the dynamics of the international system and suggests a path for future empirical and theoretical research.

STEPHAN RUSCHEWEYH, *Würzburg University, Germany*

Universally convex and starlike functions

ABSTRACT. It is well-known that the Hadamard convolution of a convex univalent functions in the unit disk with a second convex or starlike such function is also convex (starlike) univalent (Plya- Schoenberg conjecture [1958], S. R. and T. Sheil-Small [1973]). We study the same situation for so-called disk-like domains, in particular for the slit domain $\Lambda := \mathbb{C}\setminus[1,\infty)$. This leads to the notion of universally convex and starlike functions. We give a complete analytic description for these functions, and show that certain special functions, like polylogarithms, have this property. There is a strong connection with certain Pick functions whose measures are supported on the boundary of Λ. Special mention is made to the case of universally convex functions which have an analytic continuation across that boundary slit. *Joint work with A. Bakan, L. Salinas, T. Suffridge and T. Sugawa during 2006-2014.*

BARTLOMIEJ SIUDEJA, *University of Oregon*

Plate eigenvalues and higher order frames

ABSTRACT. I will discuss variational upper bounds for plate eigenvalues (biLaplacian). The sums of eigenvalues of a stretched domain will be estimated by a suitably rescaled eigenvalues of the original, highly symmetric domain (e.g. ball, equilateral triangle, icosahedron). I will formalize the "highly symmetric" assumption using tight frames and their recent generalizations called tight p-frames. The latter will be the main focus of the talk.

NAGESWARI SHANMUGALINGAM, *University of Cincinnati*

Extension of the notion of sphericalization to the metric setting

ABSTRACT. In Complex Analysis one sees the one point compactification of the complex plane as the Riemann sphere. In this talk we will give an extension of the sphericalization procedure to the setting of metric measure spaces, and discuss related geometric issues such as preservation of doubling measures and preservation of Poincare inequalities. This talk is based on joint work with Xining Li and with Davd Herron and Xiangdong Xie.

Alexander Vasiliev, *University of Bergen, Norway*

Slit holomorphic stochastic flows

ABSTRACT. We use general Loewner theory to define general slit Loewner chains in the unit disk, which in the stochastic case lead to slit holomorphic stochastic flows. Radial, chordal and dipolar SLEs are classical examples of such flows. Our approach, however, allows to construct new processes of SLE type that possess conformal invariance and the domain Markov property. The local behavior of these processes is similar to that of classical SLEs.

Matti Vuorinen, *University of Turku, Finland*

Conformal invariants, hyperbolic type metrics and quasiconformal maps

ABSTRACT. A survey of hyperbolic type metrics from the point of view of quasiconformal maps is given. Some of these metrics are quasihyperbolic metric, visual angle metric and distance ratio metric. Also some topics dealing with the case of K-quasiconformal maps when the dilatation K is close to 1 are discussed.

Shanshuang Yang, *Emory University*

Extremal length decomposition and estimates of the QED constant for Jordan domains

ABSTRACT. In this talk, we present a sharp upper bound for the QED (quasiextremal distance) constant of a Jordan domain in the plane in terms of its quasiconformal reflection constant. This confirms one conjecture about the QED constant and disproves another. One of the major ingredients used in the proof is a decomposition result for the extremal length of a curve family, which has its own interests. This is a joint work with Tao Cheng.

Organizers

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