Aspects of Fluid Dynamics

The 13th Annual Red Raider Mini-Symposium
October 25-26, 2013

Department of Mathematics and Statistics
Texas Tech University

The Red Raider Mini-Symposium Series

This series was initiated in 2001 by *Prof. Frits Ruymgaart*, a Paul Whitfield Horn Professor, who generously used his professorship's endowment to partially fund the mini-symposium's activities.

Invited lecturers at the mini-symposium include distinguished scholars who made a great impacts in the field, as well as early career mathematicians and promising young scientists. The lectures expose the audience to current research problems, as well as their solutions and applications. The topics covered by the Red Raider mini-symposia span over broad areas of pure and applied mathematics and reflect the strengths of the scientific groups in the Department of Mathematics and Statistics at Texas Tech University. In chronological order, the previous twelve symposia reflect a large diversity of research interests:

2001 *Control Theory in the Twenty-First Century*
2002 *Contemporary Algebra and Algebraic Geometry*
2003 *Mathematical and Computational Modeling of Biological Systems*
2004 *Invariant Theory in Perspective*
2005 *Geometry, Statistics and Image Analysis*
2006 *Mathematical Modeling of Novel Material and Devices*
2007 *Conformal Mapping, Circle Packing and Applications*
2008 *The Topology and Geometry of Physics*
2009 *Non-linear Analysis, PDEs and Applications*
2010 *Mathematical Modeling in Population Biology and Epidemiology*
2011 *High Level Mathematical Software for PDE's FEniCS'11*
2012 *Computational and Theoretical Challenges in Interdisciplinary Predictive Modeling Over Random Fields*

Aspects of Fluid Dynamics

The 13^{th} Annual Red Raider Mini-Symposium is entitled Aspects of Fluid Dynamics and is featuring five distinguished speakers and four early career speakers. This year, the mini-symposium is focused on the multi-facets of fluid dynamics and its applications in physics, engineering and technology. This includes both theoretical and practical research involving nonlinear analysis, partial differential equations, geometry and scientific computing.

Organizers

- Eugenio Aulisa, Texas Tech University
- Luan Hoang, Texas Tech University
- Akif Ibragimov, Texas Tech University
- Magdalena Toda, Texas Tech University

Sponsors

- Horn Professor Frits Ruymgaart, Texas Tech University
- The Department of Mathematics and Statistics, Texas Tech University

Titles and Abstracts

Michele Benzi, *Emory University*

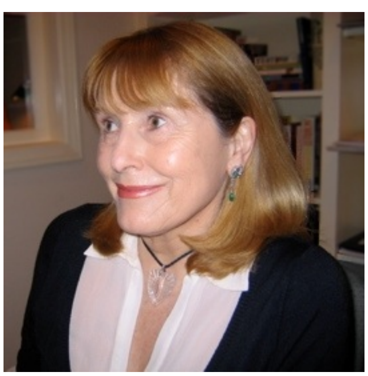
Fast Iterative Solution of Saddle Point Systems with Applications to Incompressible Flow Problems

ABSTRACT. Large linear systems in saddle point form arise in a wide variety of scientific computing problems. These include mixed formulations of second-order and fourth-order elliptic problems, incompressible fluid flow problems such as the Stokes and Oseen equations, certain formulations of the Maxwell equations, and linear elasticity problems. Often, saddle point systems arise from the use of Lagrange multipliers applied to the minimization of an energy functional subject to a linear equality constraint. The solution of saddle point problems ("KKT systems") is also of central importance in many approaches to constrained optimization, including the increasingly important field of PDE-constrained optimization.

In the first part of my talk I will discuss properties of saddle point matrices, including spectral properties relevant to the iterative solution of these systems by preconditioned Krylov subspace methods. In the second part of the talk I will give an overview of the most effective methods currently available for solving large-scale saddle-point problems, with a focus on augmented Lagrangian-based block preconditioners for incompressible flow problems (joint work with Maxim Olshanskii and Zhen Wang).

Susan Friedlander, *University of Southern California*

Active Scalar Equations and a Geodynamo Model



ABSTRACT. We discuss an advection-diffusion equation that has been proposed by Keith Moffatt as a model for the Geodynamo. Even though the drift velocity can be strongly singular, we prove that the critically diffusive PDE is globally well-posed. In contrast, the non diffusive equation is Hadamard ill-posed in Sobolev spaces. This result is proved via the construction of an unstable eigenvalue with arbitrarily large real part. The analogous construction for the diffusive equation with small coefficient of diffusivity produces strong dynamo instabilities.

Giovanna Guidoboni, *Indiana University-Purdue University Indianapolis*

Mathematical modeling of retinal blood flow and its relation to glaucoma

ABSTRACT. Glaucoma is an optic neuropathy characterized by progressive retinal ganglion cell death and structural changes to the retina and optic nerve head (ONH) ultimately leading to irreversible visual field loss. To date, elevated intraocular pressure (IOP) is the only treatable glaucoma risk factor, although there is overwhelming evidence that other factors might be involved in the disease. Many individuals with elevated IOP never develop glaucoma, and many patients develop glaucoma despite reduced IOP. In recent years, the definition of glaucoma has expanded to include vascular, genetic, anatomical, and other systemic factors. In particular, the scientific community continues to accrue evidence suggesting that alterations in ocular hemodynamics play a significant role in the OAG pathophysiology. Significant correlations have been found between impaired vascular function and both glaucoma incidence and progression; however, the mechanisms are still unclear. Clinical, population-based, and animal studies are the most popular approaches currently utilized by the scientific and medical communities to shed light on the hemodynamic contribution to the pathophysiology of glaucoma. In the recent years, we have been developing mathematical models to be used in conjunction with clinical data to unravel the mechanisms behind the observed correlations in the data. In this talk, we will present mathematical models that describe (i) the blood flow in the central retinal artery, accounting for the IOP-induced deformation of the arterial wall; (ii) the blood flow in the retinal vasculature, accounting for the interaction between the IOP mechanics on the blood vessels and the fluid-dynamics of the blood inside the vessels; (iii) the autoregulation of blood flow in the retina, accounting for the myogenic, shear-stress, CO_2 and O_2 responses. The application of such models to interpret clinical data will also be discussed.

Igor Kukavica, *University of Southern California*

Regularity of solutions for the primitive equations of the ocean

ABSTRACT. We address local and global well-posedness of solutions of the primitive equations of the ocean. First, we prove the existence of global strong solutions in the case of the Dirichlet boundary conditions and variable boundary and discuss the dissipativity of the system and uniform gradient bounds for solutions (joint work with M. Ziane). We also establish existence and uniqueness of solutions for continuous initial data (joint work with Y. Pei, W. Rusin, and M. Ziane).

Caroline Muller, *Princeton University/GFDL*

Internal waves and climate

ABSTRACT. Internal waves are gravity waves that oscillate within, rather than on the surface of, a fluid medium. These waves are ubiquitous in stratified fluids, such as the atmosphere or the ocean. In fact, in the ocean, internal waves are believed to play a key role in sustaining the deep ocean stratification and meridional overturning circulation. Understanding the ocean circulation is not only a fascinating topic in itself, but it is also a crucial ingredient for climate prediction. Internal waves impact the ocean circulation by mixing water masses where and when the waves become unstable and break. Several mechanisms can lead to the instability and breaking of the waves, and it is not known which one, if any, dominates in the ocean. In this talk, we will review the properties of internal waves and the physical mechanisms by which instabilities can develop. A heuristic model for nonlinear wave breaking and concomitant mixing will be derived. Idealized and realistic ocean settings will be investigated. Implications for the ocean circulation will be discussed.

Toan Nguyen, *Pennsylvania State University*

Stability of boundary layers in the Navier-Stokes equations

ABSTRACT. I will discuss an overview of recent results on instability of boundary layers in the Navier-Stokes equations. We'll start from the linear to nonlinear instability of the Prandtl equations in Sobolev spaces. In addition, spectral instability of the shear profiles will be discussed.

Roger Temam, *Indiana University*

Initial and boundary value problems for inviscid equations of the atmosphere and the oceans

ABSTRACT. In this lecture we will first briefly show how the inviscid two-dimensional Shallow Water (SW) equations relate to the Primitive Equations of the atmosphere and the oceans, in the context of the so-called Limited Area Models (LAM). We will then survey a number of very recent results of existence and uniqueness of solutions for linearized versions of the SW, and for the full nonlinear SW equations. Although the (linear or nonlinear) SW equations are fully hyperbolic, their time independent part can be fully hyperbolic or partly hyperbolic and partly elliptic, which gives an idea of the complexity of the problem and of the diversity of tools which are necessary. All works are in collaboration with Aimin Huang.

Alexis Vasseur, *University of Texas*

Relative entropy applied to shocks for Conservation Laws and applications

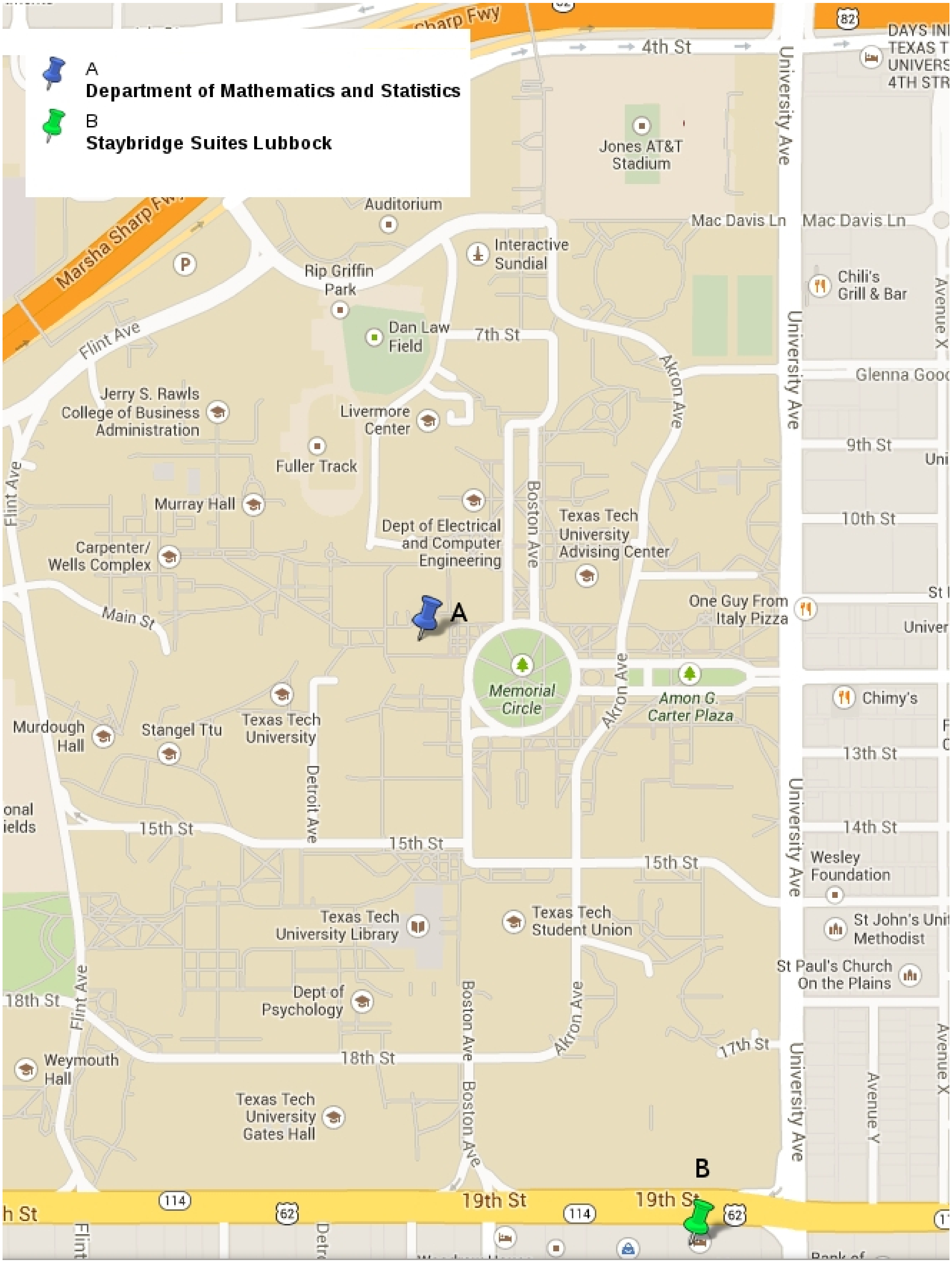
ABSTRACT. We develop a theory based on relative entropy to study the stability and contraction properties of extremal shocks of conservation laws. We will present first application of the theory to the study of asymptotic limits.

Anna Zemlyanova, *Kansas State University*

Motion of a supercavitating wedge or a hydrofoil in domains with free surfaces

ABSTRACT. The flow induced by the motion of a body and the formation of a trailing cavity is of considerable interest in marine applications such as the design and analysis of hydrofoils and marine propellers. The problem considered in this talk involves a wedge or a hydrofoil moving beneath a free surface or in a jet with a uniform speed. A trailing cavity forms behind a wedge or a hydrofoil. The cavity closure mechanism is described according to the Tulins single-spiral-vortex model. A closed form solution to the governing nonlinear boundary value problem is found by the use of conformal mappings. The double-connected flow domain is treated as the image by this map of the exterior of two slits in a parametric plane. The mapping function is constructed through the solution to two boundary-value problems of the theory of analytic functions, the Hilbert problem for two slits in a plane and the Riemann-Hilbert problem on an elliptic surface. Numerical results for the shape of the cavity and the free surface, the drag and lift coefficients, and the circulation are reported. Comparison of the numerical results with the Tulins double-spiral-vortex model is presented.

Campus Map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Department of Mathematics and Statistics
B
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University Ave
Marsha Sharp Fwy
Auditorium
Mac Davis Ln
Interactive Sundial
Rip Griffin Park
Chili's Grill & Bar
Flint Ave
Dan Law Field
7th St
Akron Ave
Glenna Goodacre
Jerry S. Rawls College of Business Administration
Livermore Center
Fuller Track
9th St
Murray Hall
Boston Ave
Texas Tech University Advising Center
10th St
Carpenter/ Wells Complex
Dept of Electrical and Computer Engineering
Main St
One Guy From Italy Pizza
Memorial Circle
Amon G. Carter Plaza
Chimy's
Murdough Hall
Stangel Ttu
Texas Tech University
Detroit Ave
13th St
14th St
15th St
Wesley Foundation
Texas Tech University Library
Texas Tech Student Union
St Paul's Church On the Plains
18th St
Dept of Psychology
17th St
Weymouth Hall
Texas Tech University Gates Hall
Avenue Y
Avenue X
19th St
114
62
82

Schedule

- Fri Oct 25, 2013
  - Morning: (Room MATH 010)
    - 8:15 am Welcome and Registration
    - 8:40 am - 9:00 am Opening remarks by Chairman Kent Pearce and Provost Lawrence Schovanec
    - 9:00 am - 9:50 am Roger Temam, Indiana University
    - 10:00 am - 10:50 am Giovanna Guidoboni, Indiana University-Purdue University Indianapolis
    - 11:00 am - 11:50 am Toan Nguyen, Pennsylvania State University
  - Afternoon: (Room MATH 010)
    - 2:00 pm - 2:50 pm Susan Friedlander, University of Southern California
    - 3:00 pm - 3:50 pm Caroline Muller, Princeton University/GFDL
    - 4:00 pm - 4:50 pm Alexis Vasseur, University of Texas
- Sat Oct 26, 2013
  - Morning: (Room MATH 010)
    - 9:00 am - 9:50 am Igor Kukavica, University of Southern California
    - 10:00 am - 10:50 am Anna Zemlyanova, Kansas State University
    - 11:00 am - 11:50 am Michele Benzi, Emory University