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Title:
Matrix Models for Semelparous Populations: Dynamics and Evolution

Abstract:

Of long standing interest in population dynamics are questions concerning the dynamic consequences and evolutionary aspects of different life history strategies. One aspect of a life history strategy is the allocation of resources to reproduction, growth, and survival and the resulting trade-offs among these. One basic strategy is semelparity, a life cycle in which an individual has only one reproductive event after which it dies (annual plants, many species of insects, etc.). In this talk I will focus on matrix models for the dynamics of semelparous populations – specifically, on nonlinear, semelparous Leslie matrix models. A basic issue for biological populations is, of course, extinction versus persistence. For a population dynamic model, this question concerns the stability and instability of zero equilibrium state, which generally loses stability as a parameter measuring reproductive output increases through a critical value (for example, as R_0 increases through 1). The bifurcation that occurs generally results in a persistence state for the population (for example, a positive equilibrium). This Fundamental Bifurcation Theorem has been proved for numerable kinds of population (and epidemiological) models. For semelparous Leslie models, however, it turns out that the bifurcation is non-generic and the Fundamental Theorem does not apply (and indeed is not in general true). As a result these models have mathematical as well as biological interest.

I will describe recent results that establish, for general nonlinear semelparous Leslie models, the existence of a dynamic dichotomy that results from the loss of stability of the extinction state. This dichotomy arises from two bifurcating branches of invariant sets that bifurcate from the extinction state as R_0 increases through 1. Mathematically, the dichotomy is between positive equilibrium states and synchronous cycles (and/or invariant loops of synchronous orbits) lying on the boundary of the positive cone. Biologically, the dichotomy is between equilibration with overlapping generations and synchronized oscillations with non-overlapping generations. I will also show that it the intensity of inter-class competition (relative to that of intra-class competition) that determines which of the bifurcating branches is attracting. I will also discuss this unusual bifurcation scenario in an evolutionary context by considering evolutionary game theoretic extensions of semelparous Leslie models.