Probability and Neurobiology

(a really presumptuous title)

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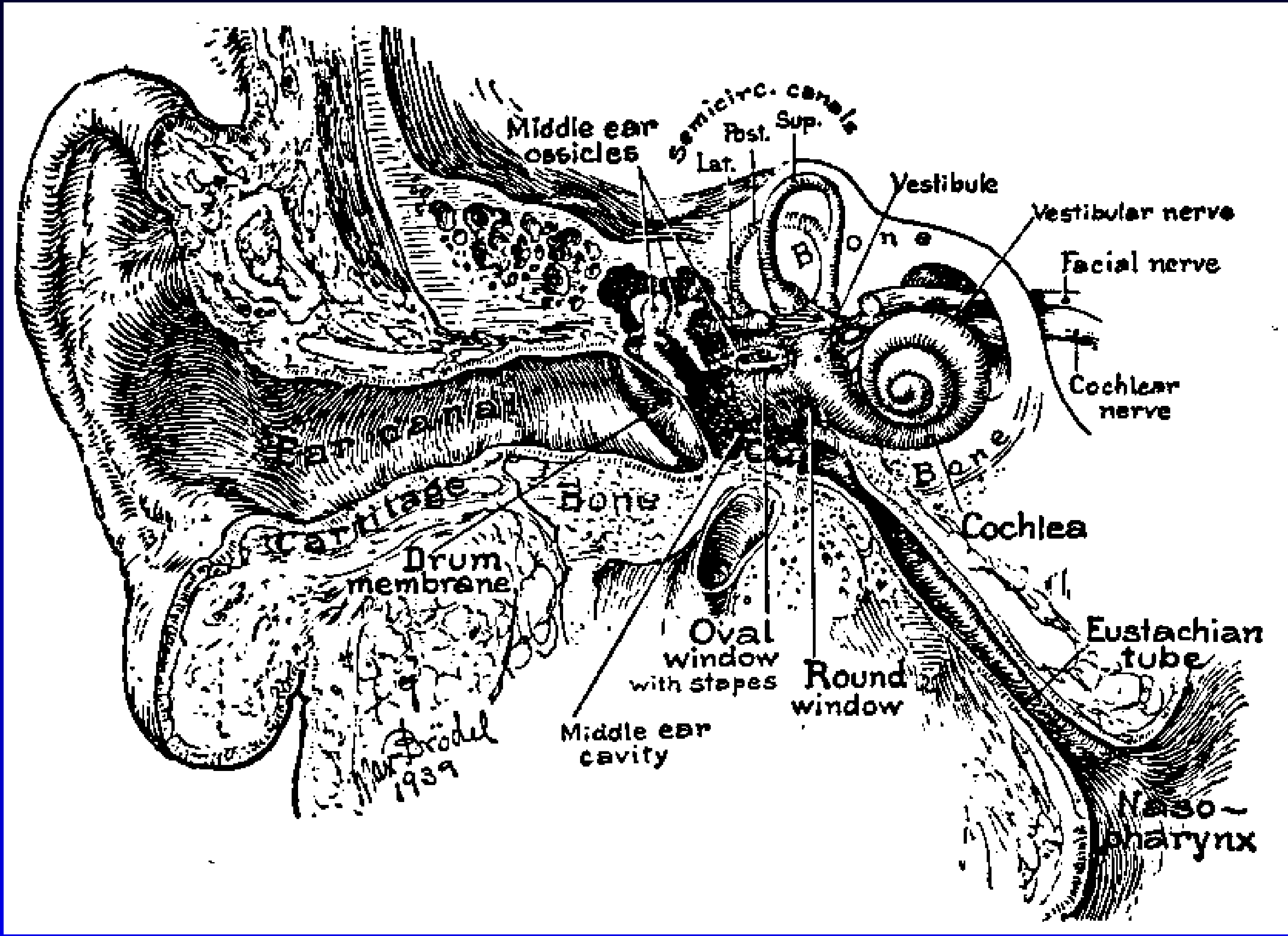
outline

- The auditory system: from the outside in.
- Latency in the auditory nerve and the lateral lemniscus.
- Formulation of a simple mathematical question.
- Computational and mathematical results.
- Current work.

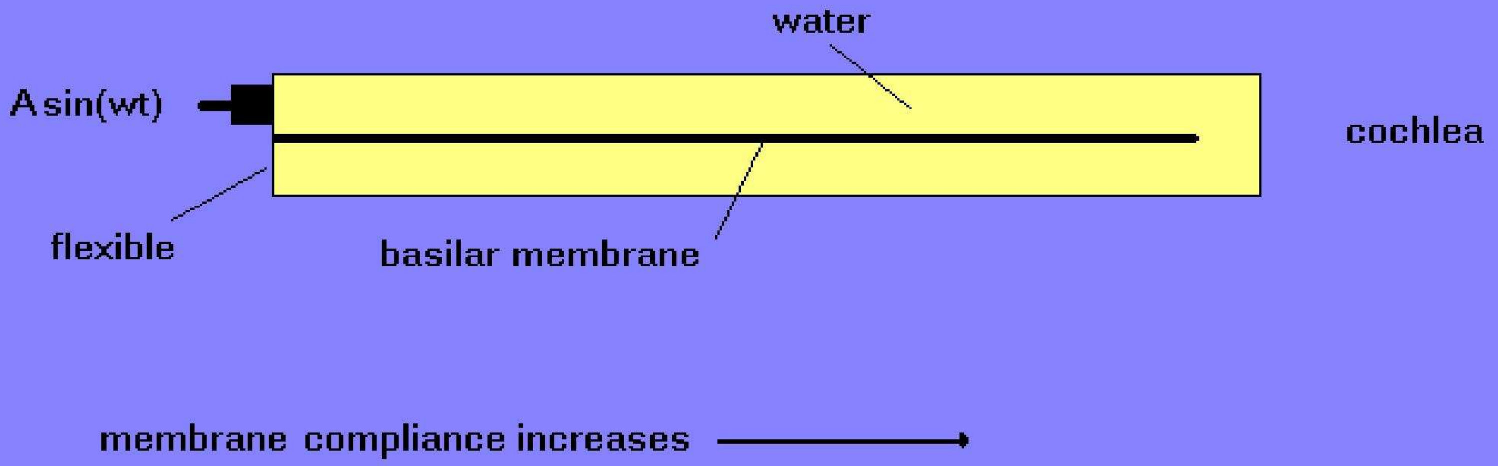
the ear



Middle ear ossicles
Semicirc. canals
Post.
Sup.
Lat.
Vestibule
Vestibular nerve
Facial nerve
Cochlear nerve
Bone
Cochlea
Eustachian tube
Naso-pharynx
Round window
Oval window with stapes
Middle ear cavity
Drum membrane
Cartilage
Bone
Max Brödel 1939

A nice mathematical problem

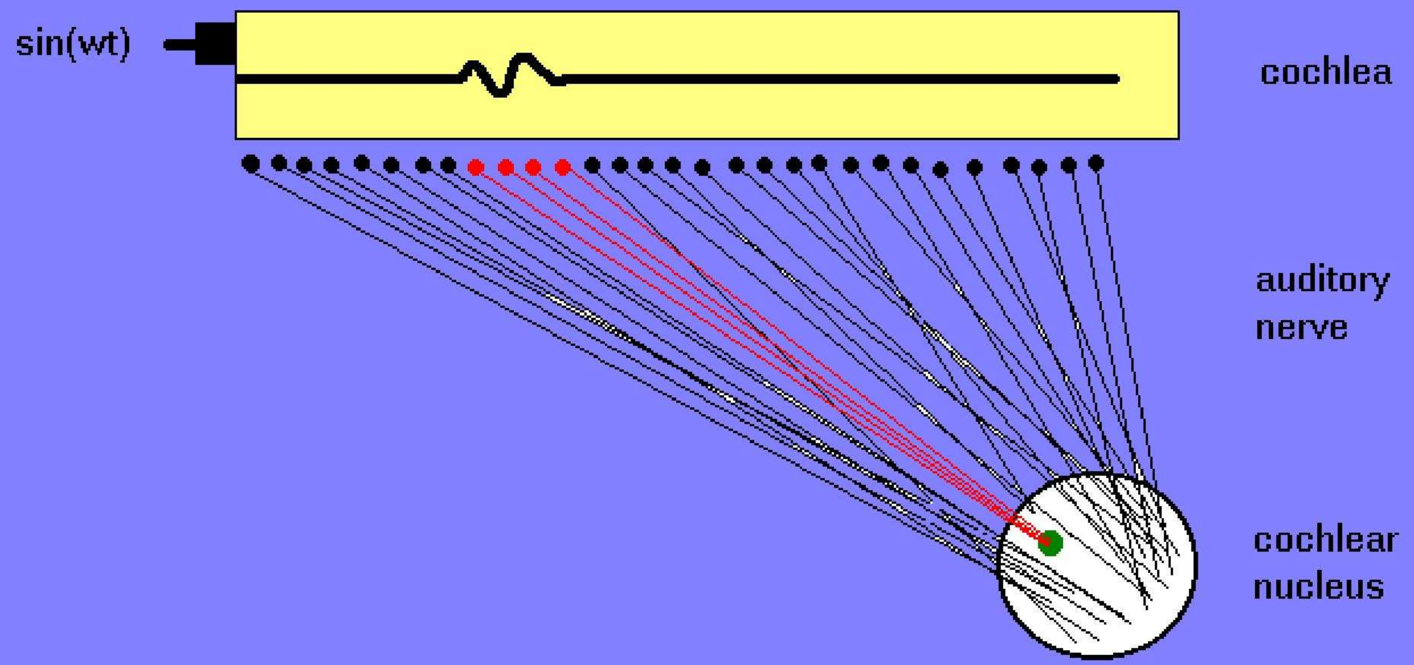
A real nice mathematical problem



G. von Bekesy, Experiments in Hearing, McGraw-Hill, 1960

C. Peskin, Lectures on Mathematical Aspects of Physiology, Lec. Appl. Math., 19, AMS, 38-69, 1981

the 8th nerve



G. von Bekesy, Experiments in Hearing, McGraw-Hill, 1960

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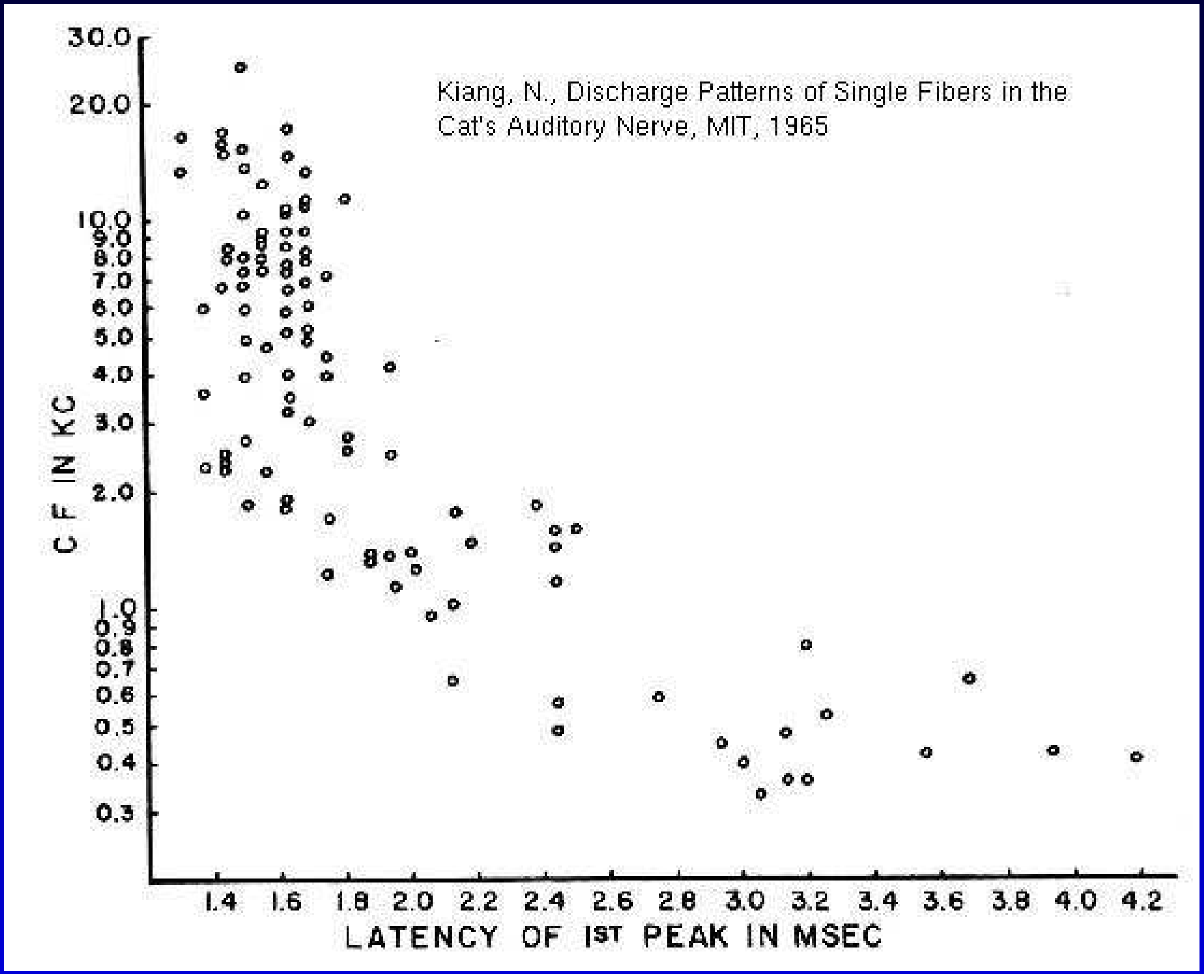
The wave delay



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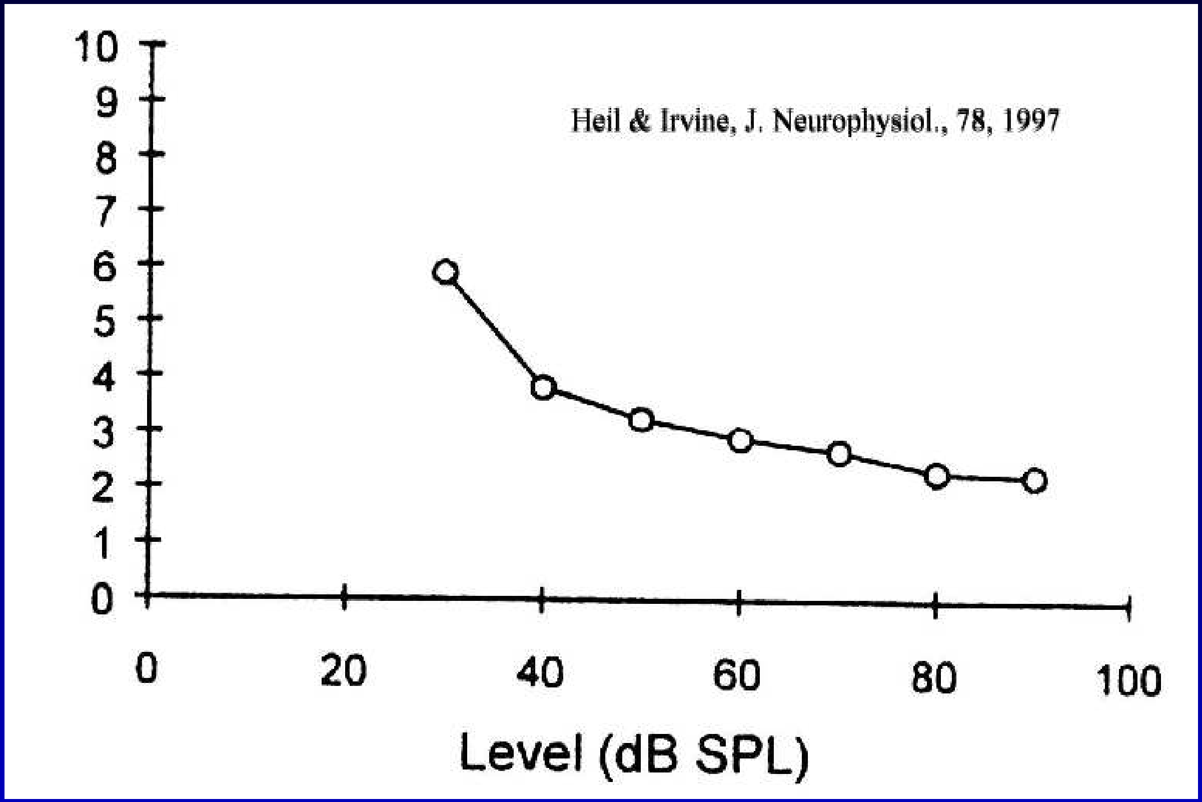
dB affects auditory nerve latency



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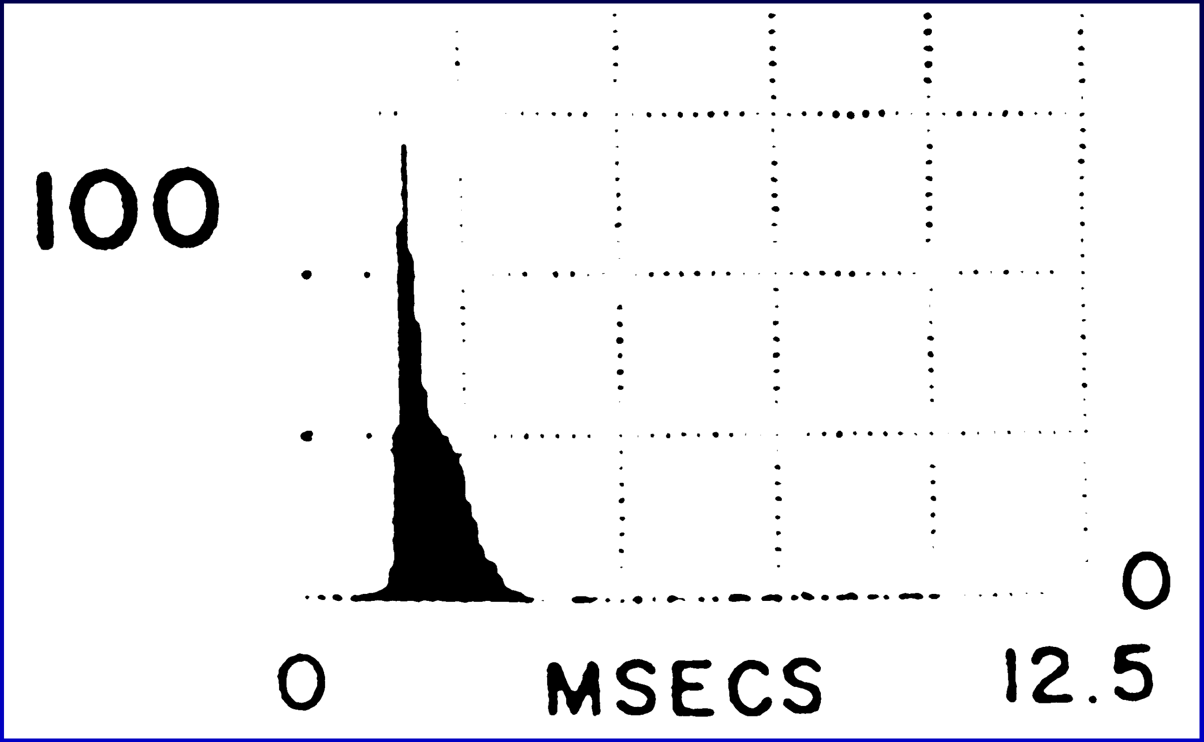
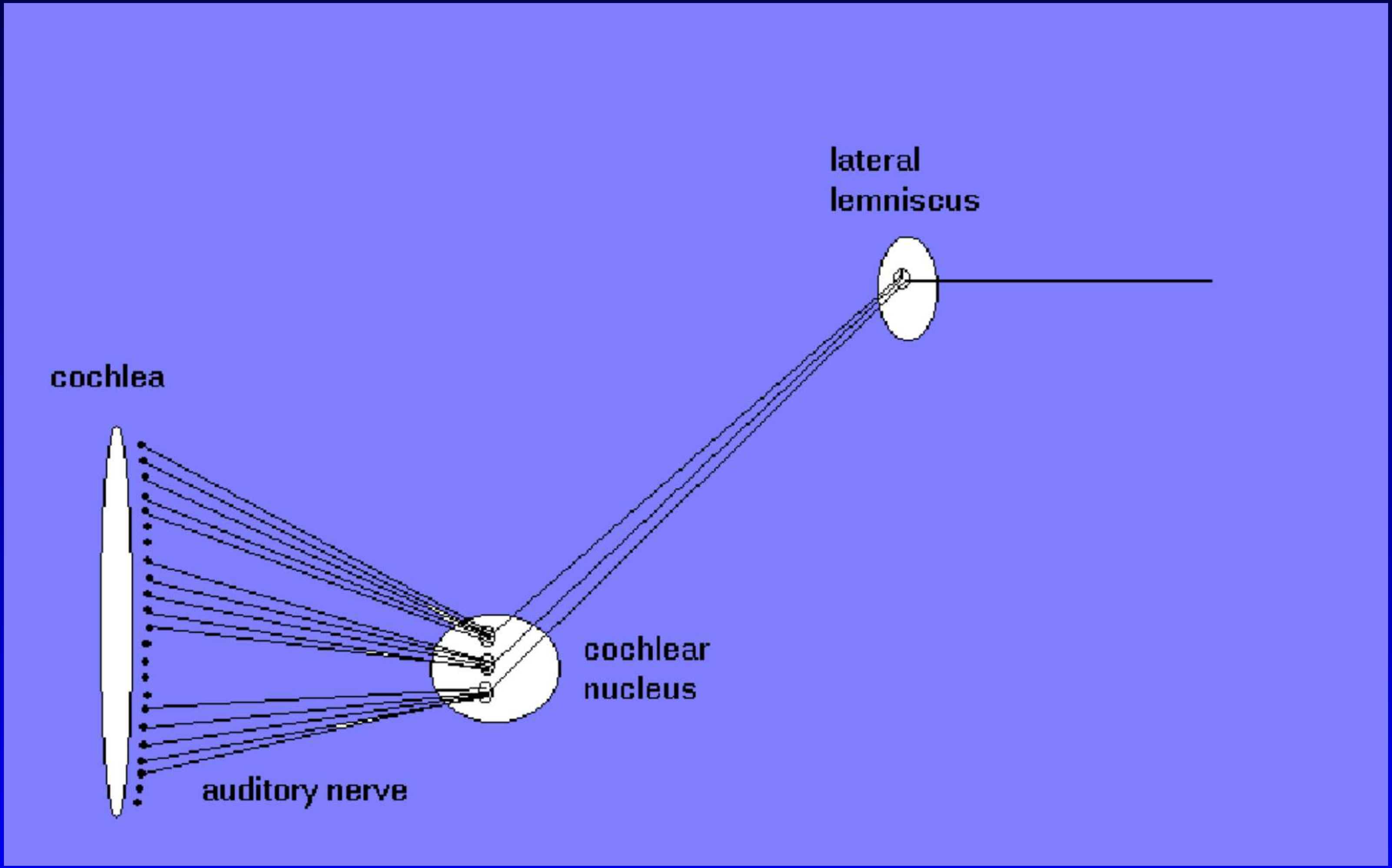
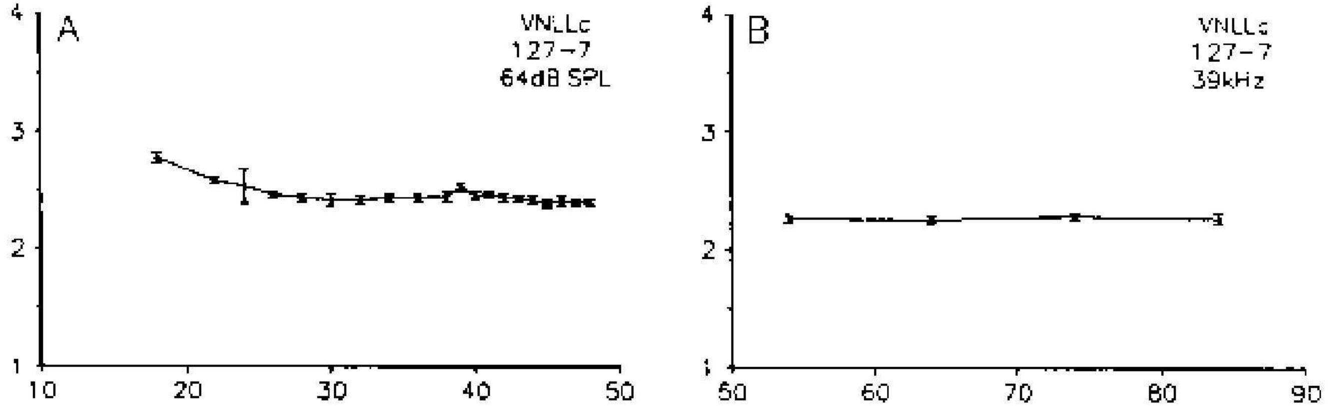
An auditory nerve histogram



From the cochlea to the Lateral Lemniscus



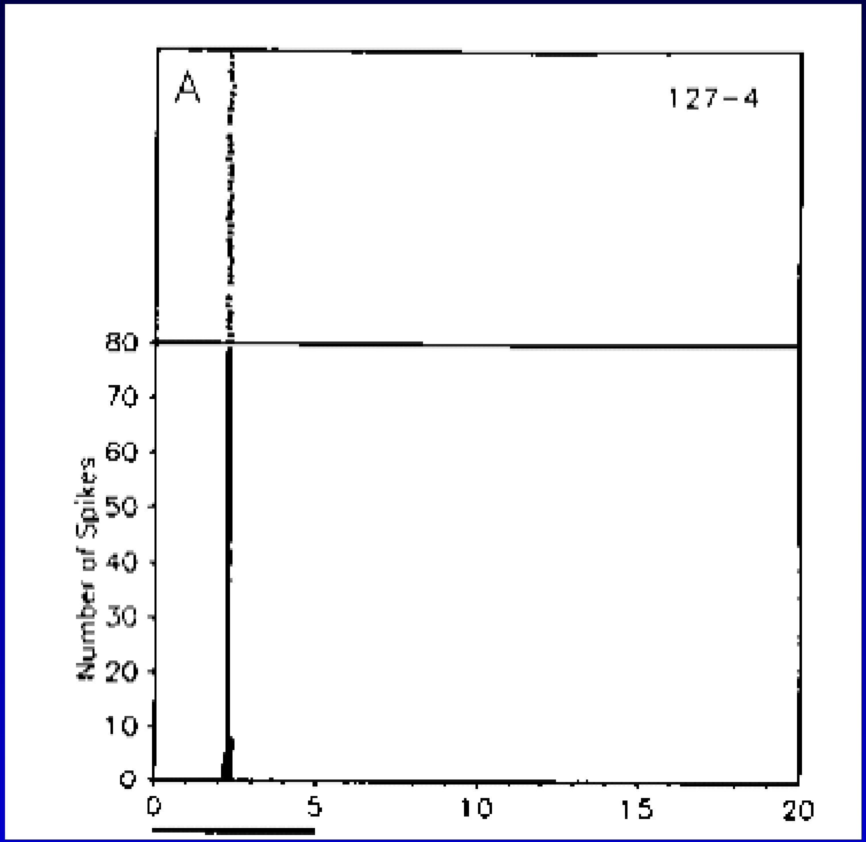
frequency and dB invariance in the lateral lemniscus



Haplea, Covey, Casseday, J. Comp. Physiol. A, 174, 1994

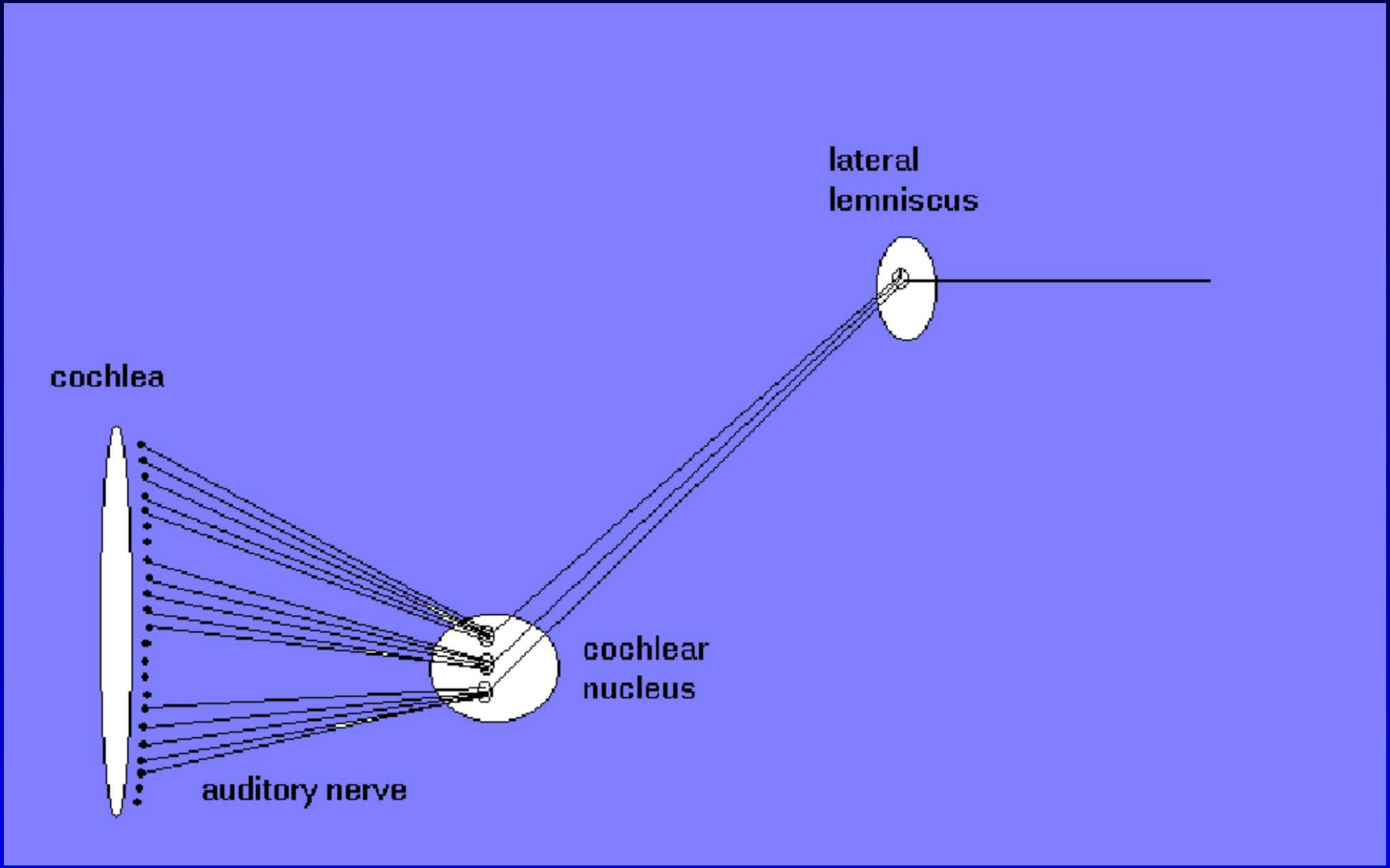
Haplea, S., Covey, E., and J. Casseday, J. Comp. Physiol., 174, 1994

A Lateral Lemniscus histogram

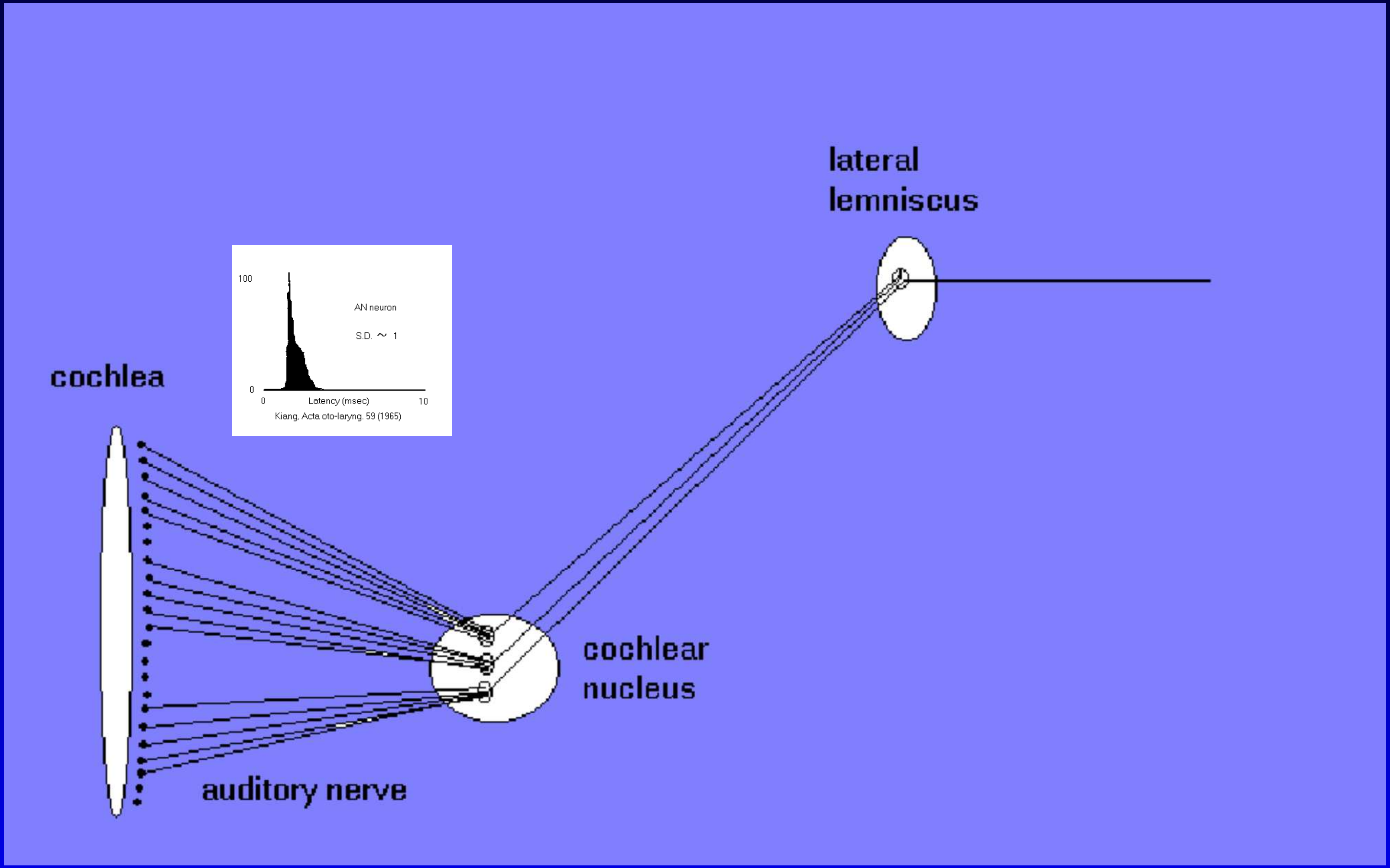


Covey, E., and J. Casseday, J. Neurosci., 11, 1991

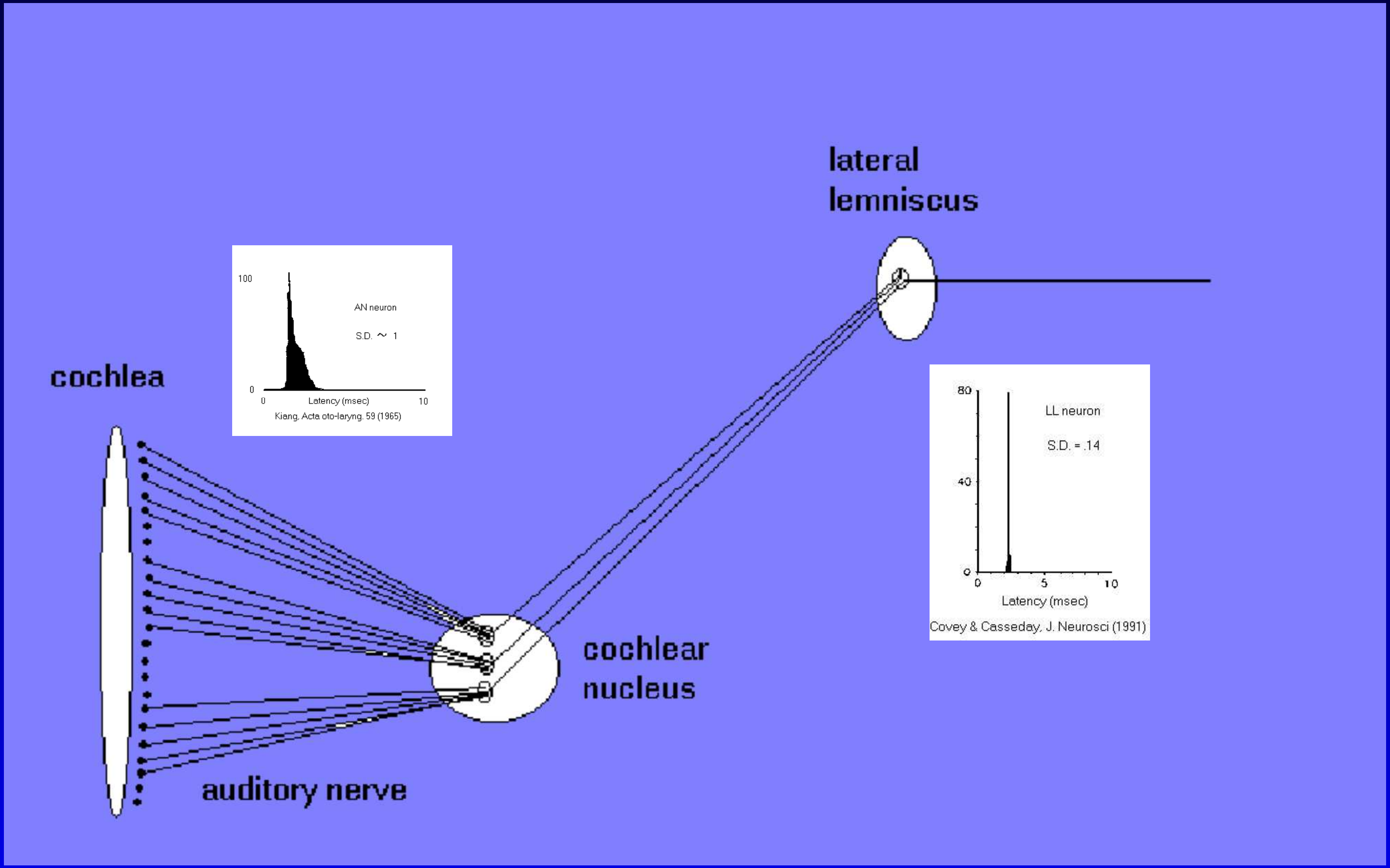
once again, from the cochlea to the lateral lemniscus



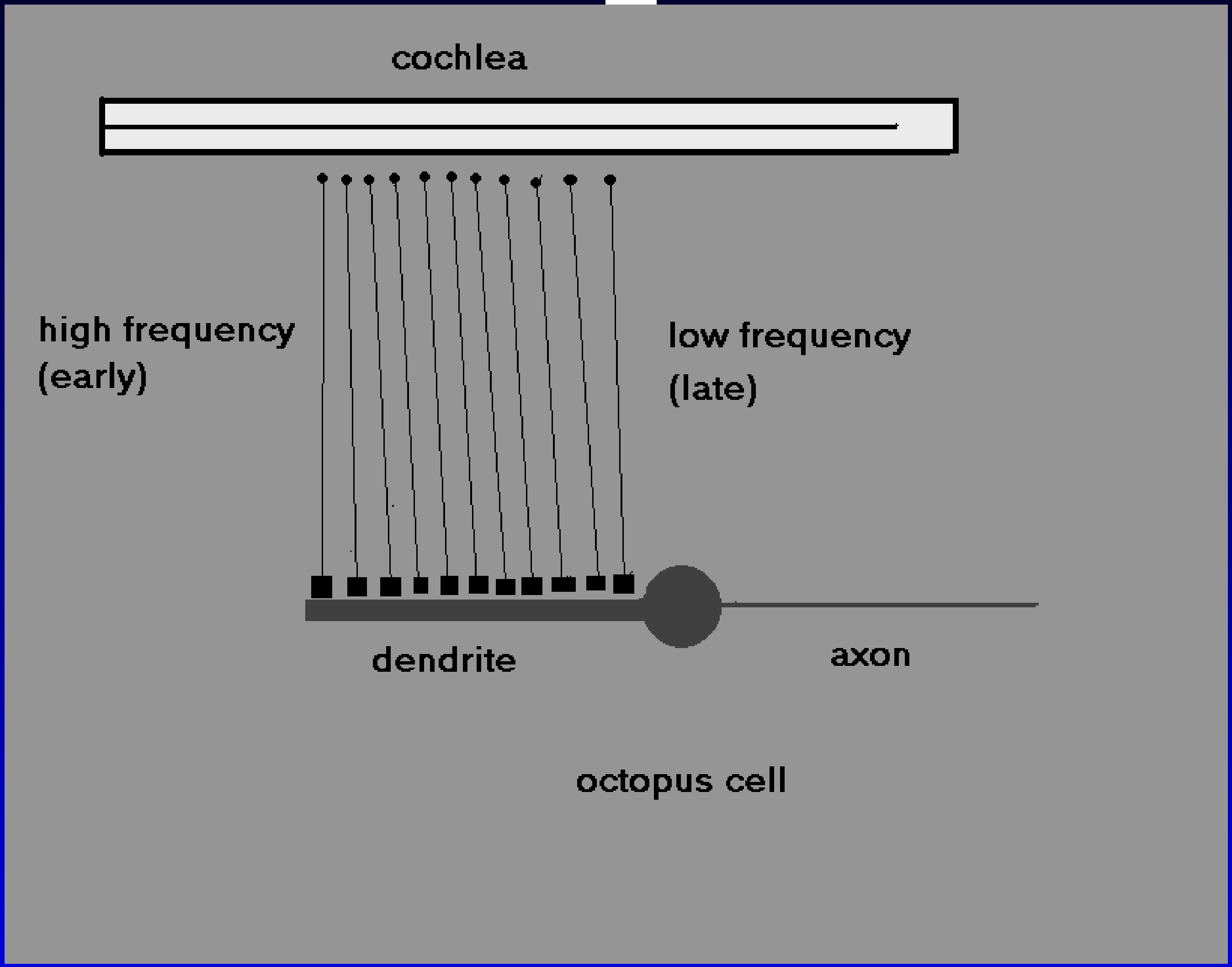
once again, from the cochlea to the lateral lemniscus



once again, from the cochlea to the lateral lemniscus



compensating for the wave delay



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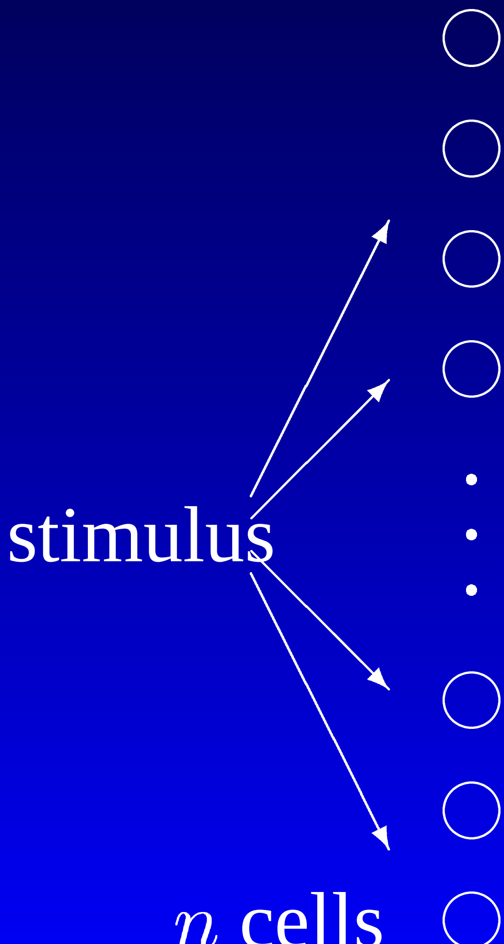
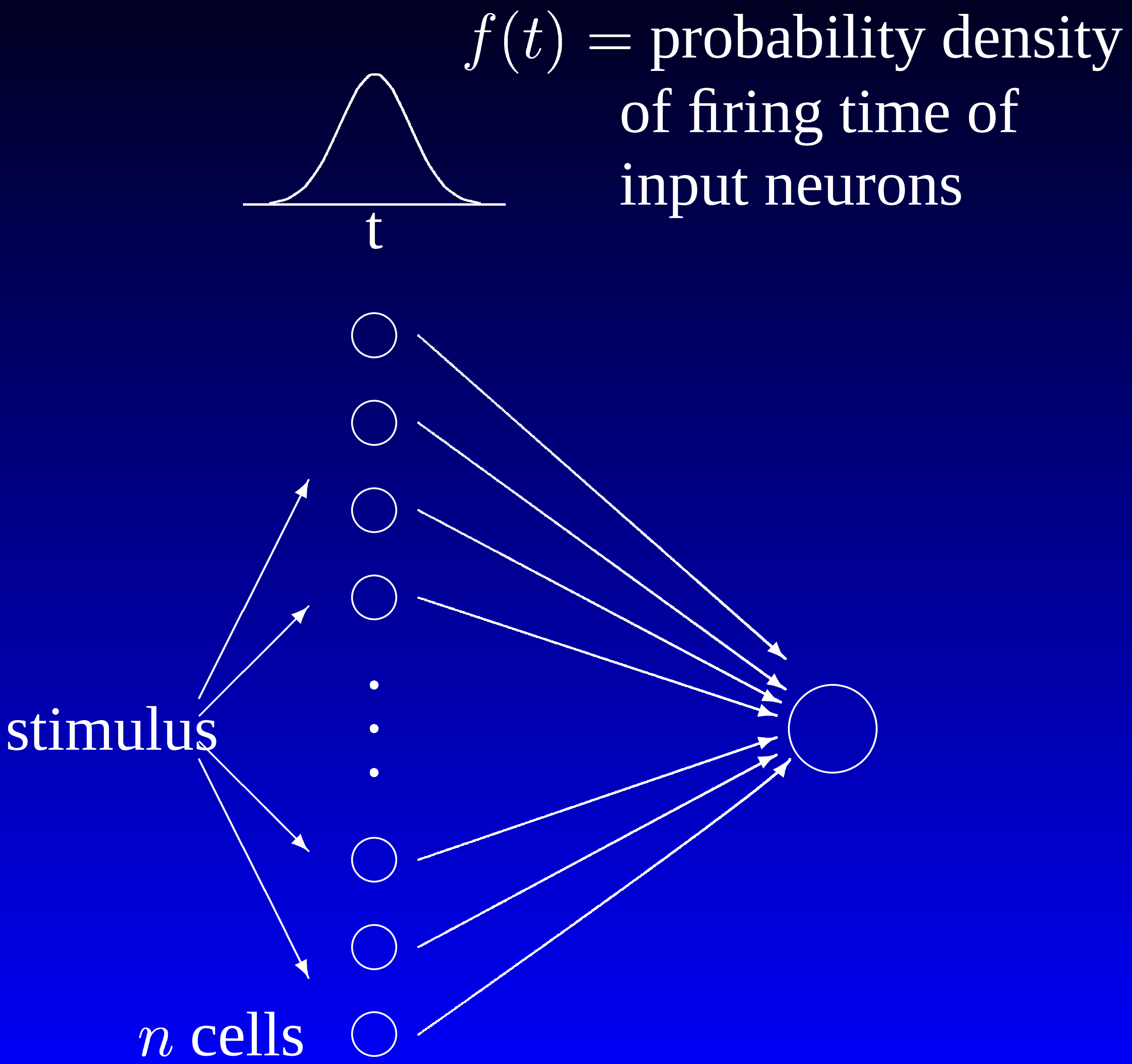
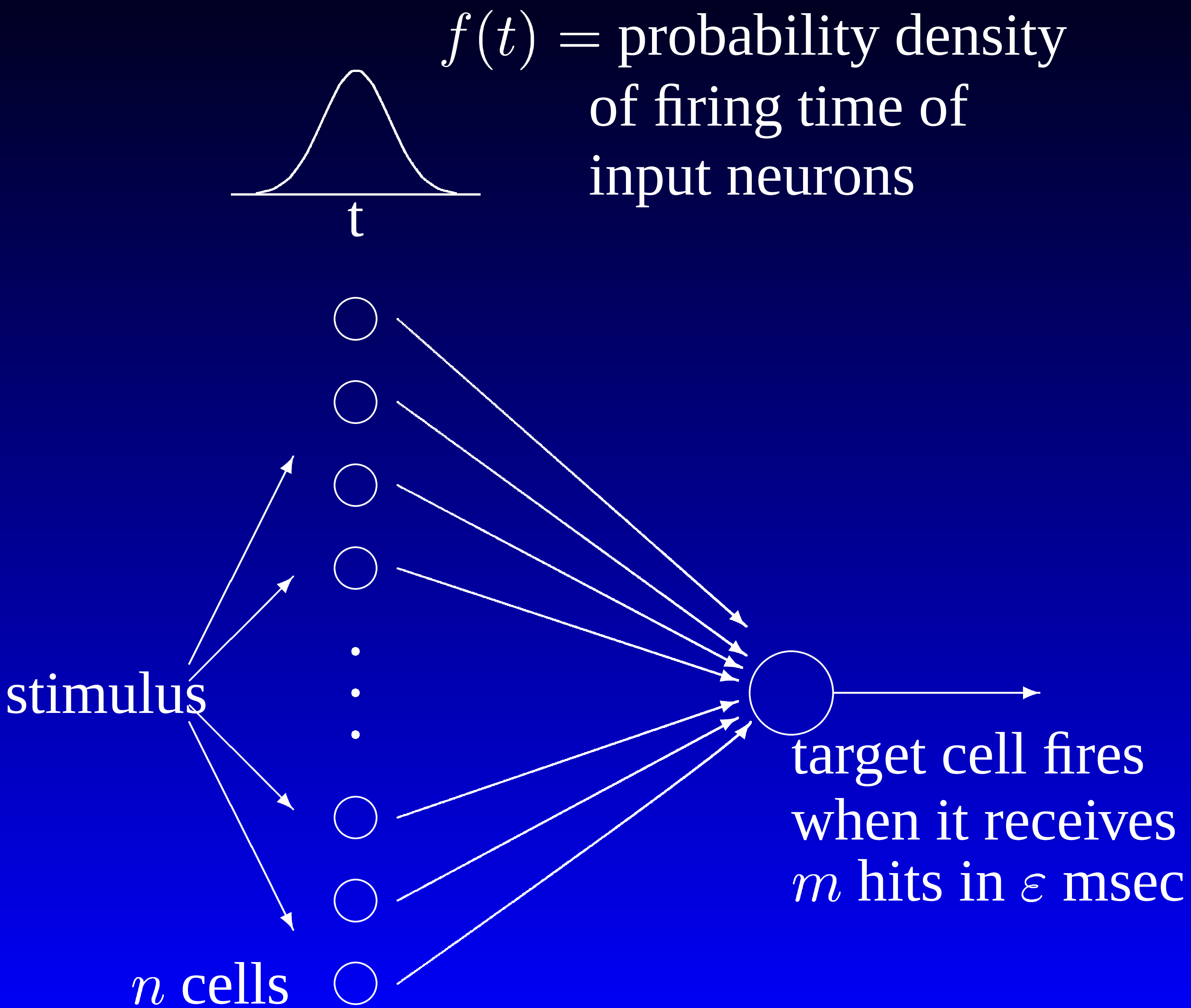
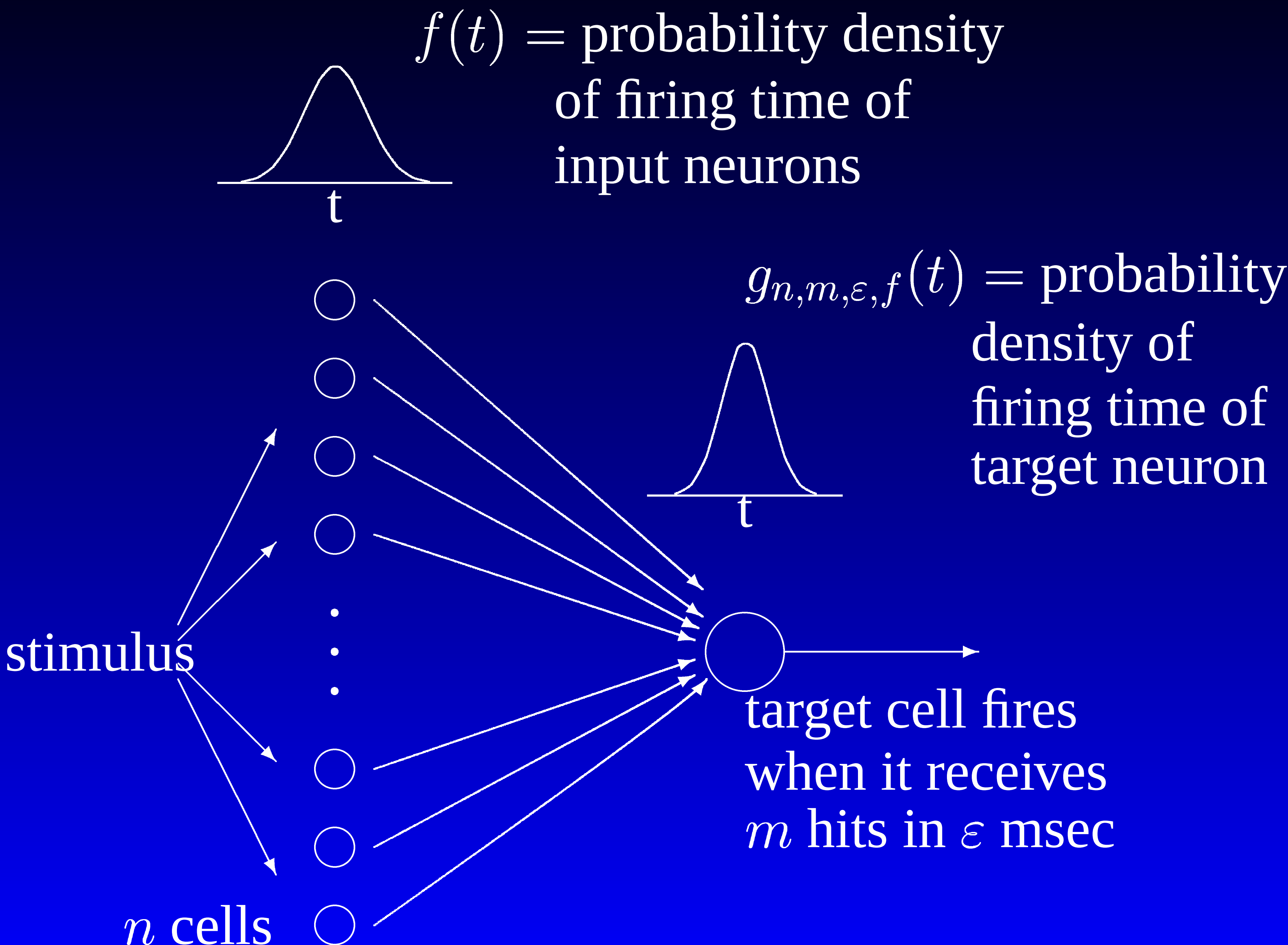
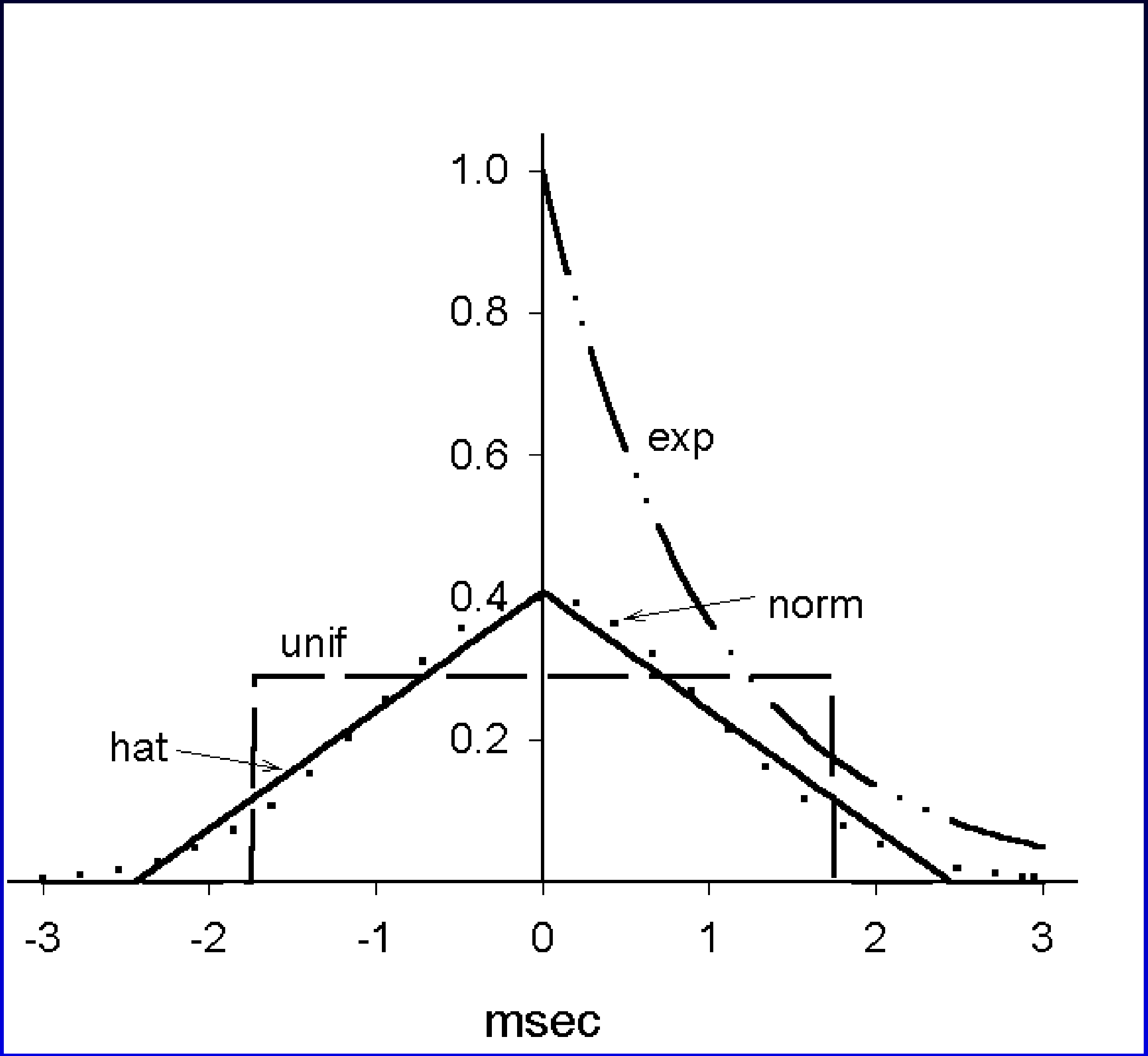

stimulus
n cells


$f(t) =$ probability density
of firing time of
input neurons
t
stimulus
n cells


$f(t)$ = probability density
of firing time of
input neurons
t
stimulus
n cells
target cell fires
when it receives
m hits in ε msec


$f(t)$ = probability density
of firing time of
input neurons
t
$g_{n,m,\varepsilon,f}(t)$ = probability
density of
firing time of
target neuron
t
stimulus
n cells
target cell fires
when it receives
m hits in ε msec

experimental densities

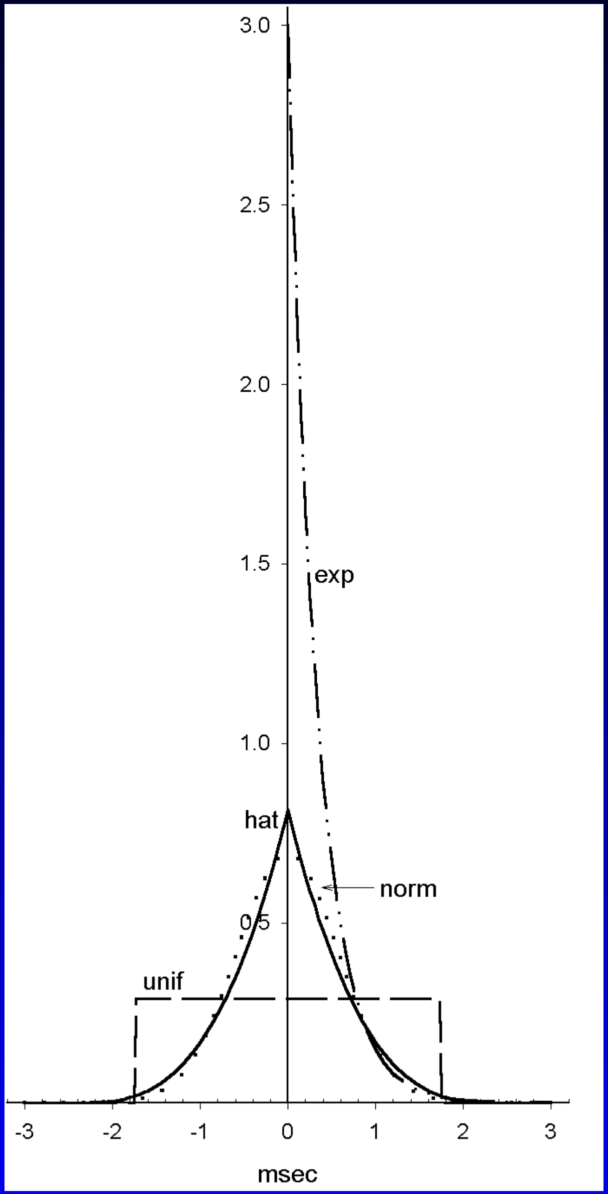


Theorem: Let f be a density and let $f \in L^{m+1}$, then

$$g_{n,m,\varepsilon,f} \xrightarrow{L^1} \frac{f^m}{\int f^m}$$

- If, in addition, we require f to be left-continuous, then the convergence is also pointwise.
- If, in addition, we require f to be uniformly continuous, then the convergence is also uniform.
- If, in addition, we require f to be in the weighted L^p space $L^{m+1,1}$, the standard deviations converge.

cubing the densities



$\varepsilon = 0$

$$\sigma_{n,m,0} = 1, \quad \text{if } f \text{ is uniform.}$$

$$\sigma_{n,m,0} = \frac{1}{m}, \quad \text{if } f \text{ is exponential.}$$

$$\sigma_{n,m,0} = \frac{1}{\sqrt{m}}, \quad \text{if } f \text{ is normal.}$$

$$\sigma_{n,m,0} = \frac{\sqrt{12}}{\sqrt{(m+2)(m+3)}}, \quad \text{if } f \text{ is hat.}$$

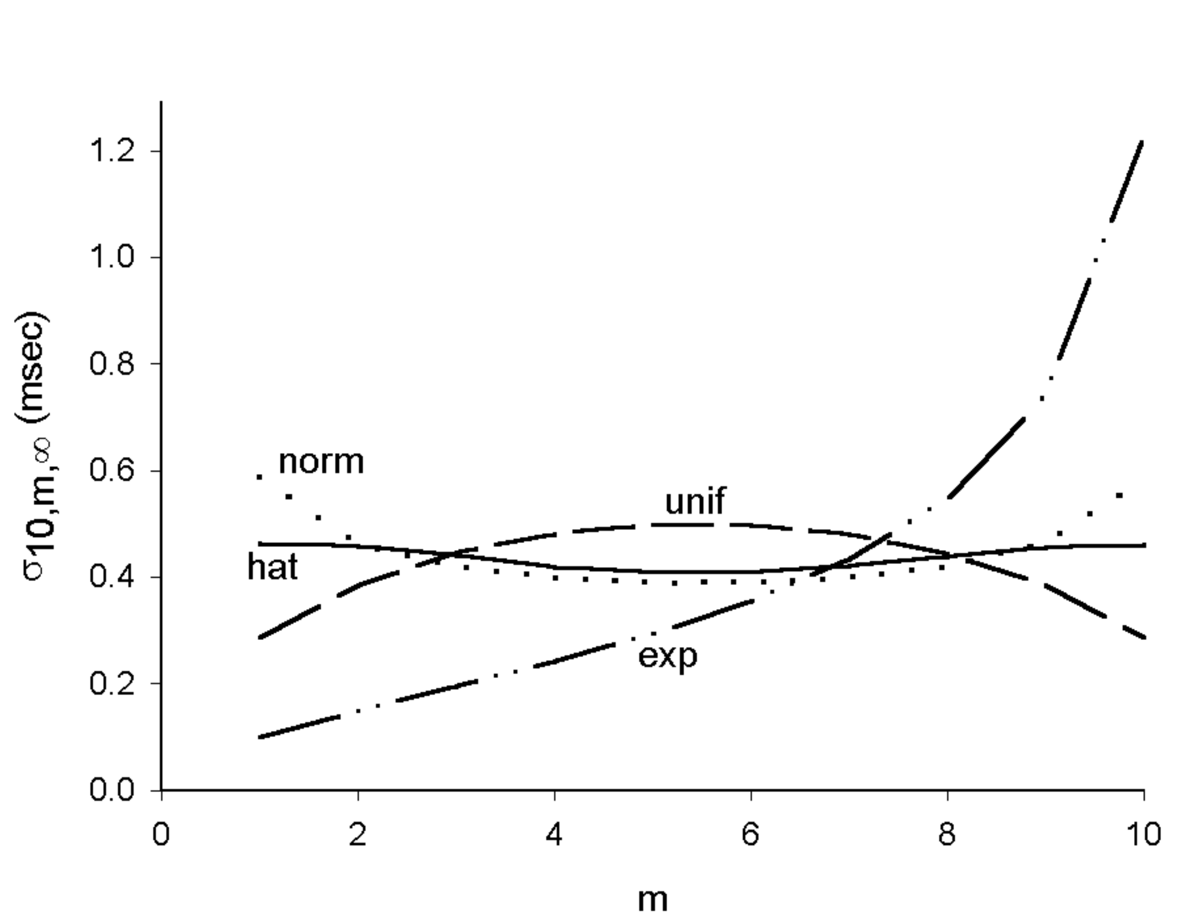
$\varepsilon \to \infty$

$$g_{m,n,\varepsilon,f}(t) \longrightarrow n! f(t) \frac{F(t)^{m-1}}{(m-1)!} \frac{(1-F(t))^{n-m}}{(n-m)!}$$

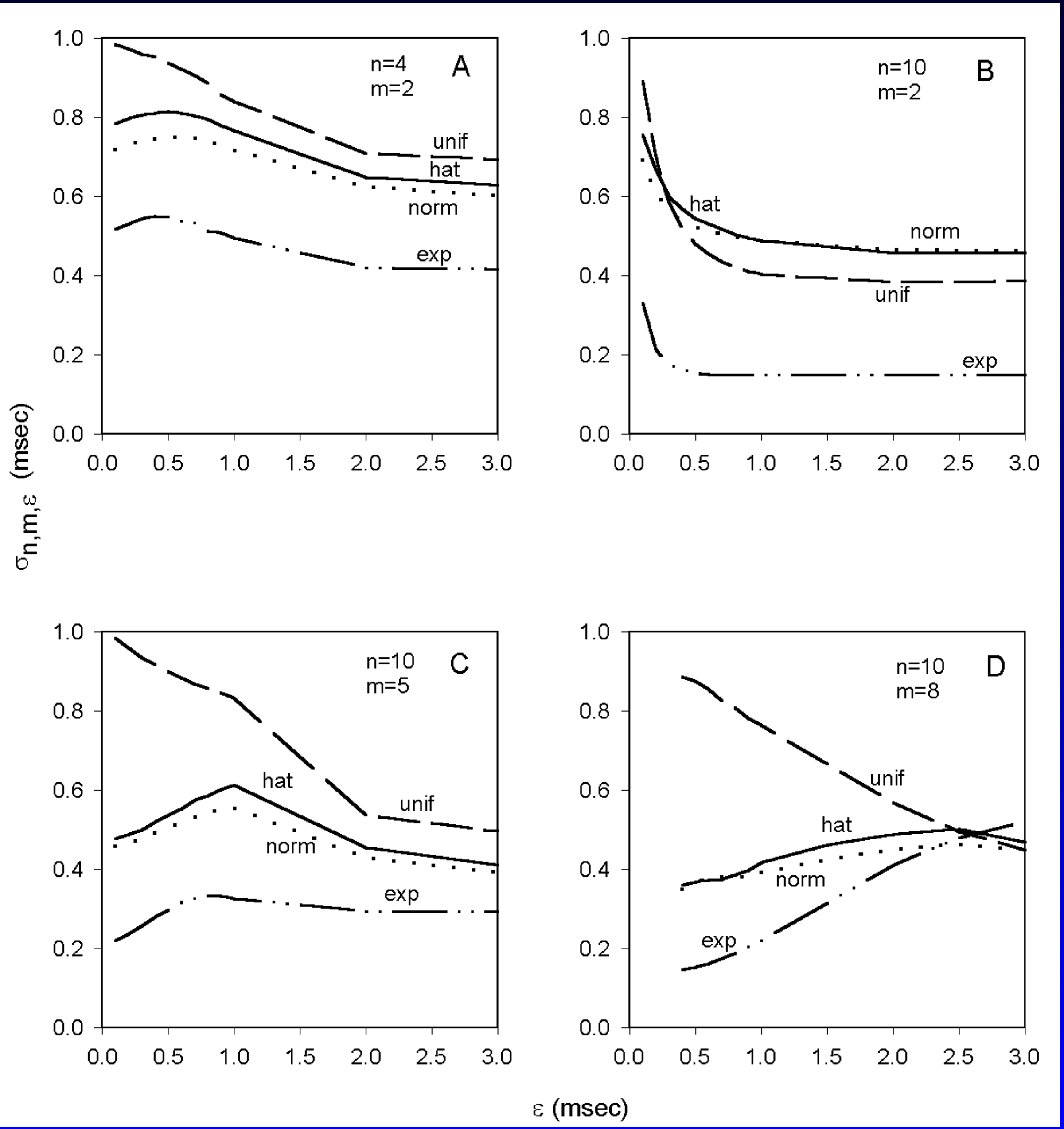
The target cell fires when the mth hit arrives.

Order Statistics.

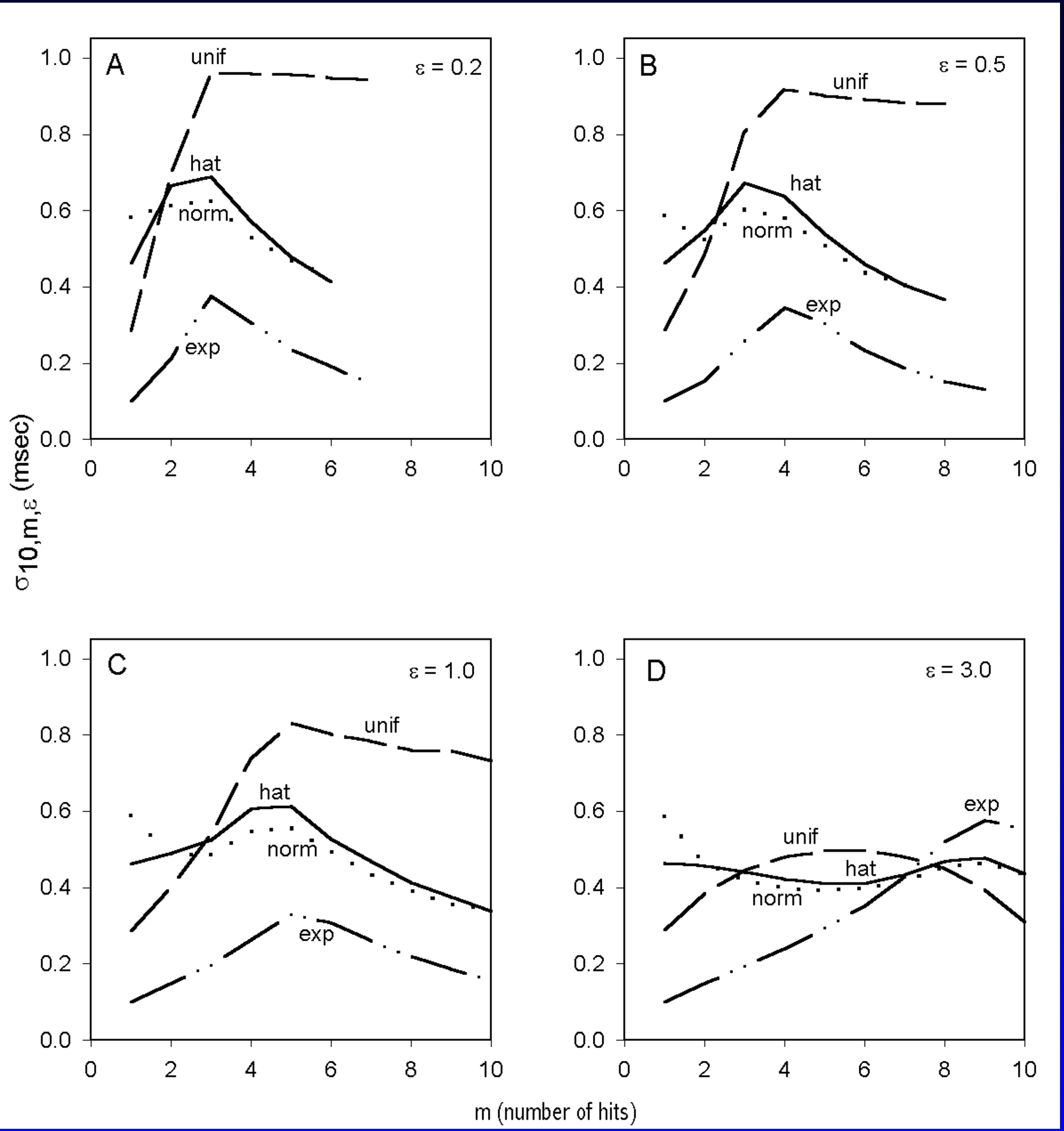
epsilon = ∞



dependence on epsilon



dependence on m



current work

- Do neurons have time windows?

current work

- Do neurons have time windows?
- other CNS systems

current work

- Do neurons have time windows?
- other CNS systems
- error correction, information sharpening

Why is the CNS so hard?

- Single neurons?

Why is the CNS so hard?

- Single neurons?
- Large scales?

Why is the CNS so hard?

- Single neurons?
- Large scales?
- Individual differences!

Why is the CNS so hard?

- Single neurons?
- Large scales?
- Individual differences!
- Ever changing!

mathematical biology

- Purpose?

mathematical biology

- Purpose?
- How?

mathematical biology

- Purpose?
- How?
- Who? (we're good at it!)

mathematical biology

- Purpose?
- How?
- Who? (we’re good at it!)
- Thanks!

References

Precision of Neural Timing: Effects of Convergence and Time-Windowing.
M. Reed, J. Blum, C. Mitchell, J. Comp. Neurosci. 13:35-47 (2002).

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C. Mitchell, submitted.