

# An Empirical Case Study of Factor Alignment Problems Using the USER Model

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The practical issues that arise due to the interaction between three principal players in any quantitative strategy—namely, the alpha model, risk model, and constraints—are collectively referred to as factor alignment problems (FAPs). Examples of FAPs include risk-underestimation of optimized portfolios, undesirable exposures to factors with hidden and unaccounted systematic risk, consistent failure in achieving ex ante performance targets, and inability to harvest high-quality alphas into an above-average investment return. Loosely speaking, FAPs symbolize the gut-wrenching ex post feeling a portfolio manager (PM) often experiences that prompts him/her to ask questions such as “did the risk model eat my alpha,” “should I have included that alpha factor in my risk model,” or “how can I transform these high IC alphas into high performance portfolios”? This article concerns empirical illustration of various facets of FAPs using the U.S. Expected Return (USER) model of Guerard et al. [2012].

Unlike previous studies on FAPs that are either based on simulated returns or a black-box expected returns model, we leverage the detailed knowledge of the USER model to create an insightful narrative. We show that optimal portfolios constructed using the USER model without taking into account the misalignment issues betray typical symptoms of FAPs and have exposure to certain hidden

systematic risk factors that are not accounted for during portfolio construction. We trace the origins of these latent systematic risk factors to the constituent factors of the USER model and the turnover constraint. Finally, we leverage our understanding of the alignment issues to propose an alternative portfolio construction methodology that directly addresses FAPs. Using the proposed methodology not only gives unbiased risk forecasts but also improves the ex post performance in a statistically significant manner.

Several authors have examined FAPs recently and have proposed various solution techniques. Saxena and Stubbs [2010a] conducted an empirical case study to understand the risk underestimation problem, a prominent symptom of FAPs. The authors used real-world data and a battery of backtests to demonstrate the perverse and pervasive nature of FAPs. They demonstrated that all optimized portfolios share a common property, namely, they have exposure to certain kinds of latent systematic risk factors that are uncorrelated with factors of the risk model that was used to generate them. Ceria et al. [2012] examine potential sources of the mentioned systematic risk factors and suggest that proprietary definitions of certain style (B/P, E/P, and so on) and technical factors can introduce them. Lee and Stefek [2008] illustrate a similar idea by using two different definitions of a momentum factor to define alpha and risk factors

and argue that the optimizer is likely to load up on the difference between the two, thereby taking unintended bets. Finally, Saxena and Stubbs [2010b] present detailed analysis of the so-called alpha alignment factor (AAF) approach. Originally intended to solve only the risk underestimation problem, AAF soon emerged to be an effective remedy to FAPs. The authors show analytically that using the AAF approach not only removes the bias in risk prediction but also improves the ex post performance.

One of the fundamental obstacles in researching FAPs is the proprietary nature of alpha models that are used in practice. While most quantitative managers use some variant of growth, value, momentum, or earnings consensus factors to define their expected return forecasts, the specific nature of their alpha factors and calibration methodology remains a closely guarded secret. This severely handicaps researchers' ability to formulate a meaningful narrative on FAPs for two reasons.

First, in the absence of a detailed alpha model, it is impossible to provide insights into the sources of latent systematic risk. For instance, Saxena and Stubbs [2010a] used a test bed of real-world expected returns to demonstrate the existence of latent systematic risk factors and provide an empirical explanation for the risk underestimation problem. Their findings, however, could have been substantially more impactful if they could have traced the latent systematic risk to specific components of the expected return model. In this article, we provide that missing link by leveraging the detailed knowledge of the USER model. Besides demonstrating the existence of latent systematic risk, we also identify components of the USER model (BP, REP, PM, and so on) and constraints (turnover constraint, long-only constraint) that contribute to unaccounted systematic risk. In fact, we compute the amount of systematic risk that goes undetected by virtue of exposure of the optimal portfolio to each component of the USER model. We aggregate all of these constituents in a systematic manner to devise an adjusted risk estimate that tracks the realized risk of the portfolio in an unbiased fashion, thereby eliminating the bias in risk prediction. To the best of our knowledge, such a detailed structural analysis of latent systematic risk has never been attempted before.

Second, several authors have conjectured that different definitions of style factors could be a major contributor to FAPs (Ceria et al. [2012]; Lee and Stefek [2008]). Inability to access proprietary definitions of

style factors such as momentum, adjusted B/P or E/P, earnings consensus, and so on, naturally makes it difficult to provide an empirical validation of these conjectures. This article goes a long way in filling that gap. We give detailed cross-sectional and time-series analytics associated with factors in the USER model to confirm that minor differences in style definitions can introduce latent systematic risk in the resulting portfolios. For instance, our results indicate that the residuals obtained by regressing the B/P factor in the USER model against factors in Axioma's fundamental medium-horizon risk model (US2AxiomaMH) are statistically significant in 50% of the periods and have significant systematic risk exposure, despite being uncorrelated with all the factors in the US2AxiomaMH risk model. Since the value factor in US2AxiomaMH is also derived from the B/P valuation ratio, it follows that adjustments that were made to B/P before being incorporated in US2AxiomaMH and the USER model, respectively, were significant enough to introduce exposure to latent systematic risk factors.

The rest of the article is organized as follows. In the following section, we review some of the key characteristics of FAPs and the USER model. We show that constituent factors in the USER model are only partially explained by factors in the risk models that we use in our experiments; this mirrors the situation faced by most quantitative managers and adds practical relevance to the results presented in the following sections. We then lay out our experimental setup and present computational results obtained without making adjustments for FAPs. Unsurprisingly, the resulting portfolios display the quintessential symptom of FAPs, namely, downward bias in risk prediction. We raise several pertinent questions setting the stage for the following sections to answer them. The next section contains the most important contributions of the paper: We present a detailed analysis of latent systematic risk exposures of the optimal portfolios generated using the USER model. We give cross-sectional and time series statistics to quantify the extent of systematic risk that goes undetected in the construction of these portfolios. We leverage the insights garnered previously to propose an effective remedy for FAPs using the AAF methodology. We demonstrate that using the mentioned methodology not only removes the bias in risk prediction, but also enhances ex post performance in a statistically significant manner. We close the paper with concluding remarks. Throughout this article, we use the phrases alpha and expected return synonymously.

## FAPs AND THE USER MODEL

In this section, we provide a high-level discourse on the sources and effects of FAPs. For a detailed investigation of these issues, see Renshaw et al. [2006]; Lee and Stefek [2008]; Saxena and Stubbs [2010a,b]; and Ceria et al. [2012].

The most common source of FAPs is misalignment between various components of an equity strategy. For instance, consider a growth portfolio manager who is using the earnings yield (E/P) factor with a negative weight in his expected return model and the “growth” factor, defined as the annualized growth in earnings per share (EPS), in the risk model. Under the assumptions of the classical single-stage dividend discount model (DDM), constant payout ratio, and risk-free interest rate, the mentioned growth and E/P factors should be identical and, hence, completely aligned with each other. Unfortunately, the practical world is more complicated and twisted than that assumed by the DDM model, which introduces incongruity between the alpha factor (negative E/P) and the risk factor (“growth”).

Besides, alpha modelers often make adjustments to the accounting data to express their proprietary views on various aspects of the underlying company. For instance, they might decide to capitalize R&D costs that are usually expensed under most accounting standards, make goodwill adjustments to alleviate the effects of past overvalued acquisitions, incorporate pension assets/liabilities to compute the “actual” leverage of the firm, and so on. These are manager-specific adjustments that improve directional forecasts but do not necessarily add value in explaining cross-sectional dispersions; understandably third-party risk-model vendors usually do not resort to such massaging of B/S, I/S, C/F data. Naturally, this introduces misalignment between the alpha and risk factors. Misalignment can also arise due to differences in definition of technical factors (short/medium-term momentum), calibration procedures (choice of asset universe), and classification systems (GICS or ICB).

Most quantitative strategies employ additional constraints that model aspects of the investment process not captured by alpha or risk models. These might include Investment Policy Statement (IPS)–motivated limits on exposures to specific securities, turnover considerations, liquidity concerns, tax-related exposures, negative constraints due to ethical or moral considerations (e.g., SRI), and so on. Each one of these constraints alters

the de facto alpha (referred to as the implied alpha) that is used to derive optimal holdings, thereby effectively introducing hidden factors in the alpha model. If these hidden factors are not aligned with the risk factors, FAPs are inevitable. For instance, the risk model might use daily bid-ask spreads to model the liquidity factor, while the PM can use average daily volume (ADV) to control exposure to liquid securities. Despite being highly correlated, these two notions of liquidity are not identical, resulting in misalignment, congruence between the alpha and risk factors notwithstanding.

Due to various sources of misalignment the (implied) alpha used in a quantitative strategy often has a component that is uncorrelated with the risk factors included in the risk model. In an ideal world wherein the risk model captures *all* sources of systematic risk, the mentioned uncorrelated component should have only idiosyncratic risk. Of course, capturing all sources of systematic risk is too ambitious a goal for any realistic risk model, and most architects of such models content themselves by capturing the key systematic risk factors (industry factors, growth, value, momentum, and so on) and dropping the rest of them in the interest of stability of the covariance matrices, estimation error minimization, avoiding data mining biases, and so on. While the risk model, and by association the optimizer, perceives no systematic risk in the uncorrelated component, it might thus have latent systematic risk that goes undetected during the portfolio construction process. Consequently, the optimizer loads up on the uncorrelated component, resulting in a skewed composition of optimal holdings and taking inadvertent exposure to hidden systematic risk factors. This naturally introduces downward bias in the ex ante risk prediction of optimal portfolios and other well-known symptoms of FAPs. Detailed empirical illustration of this phenomenon constitutes the emphasis of this article; we use the USER model (see Guerard et al. [2012a]) and Axioma’s medium-horizon fundamental (US2AxiomaMH) and statistical (US2AxiomaMH-S) risk models to pursue our goal. The USER model is used to define expected return forecasts, whereas Axioma’s risk models are used to obtain ex ante risk predictions during portfolio construction.<sup>1</sup>

We refer the reader to Guerard et al. [2012a] for details on the USER model. We excerpt its key features in this section to keep the manuscript self-contained. The USER model is a multifactor security selection model given by,

$$TR_{t+1} = a_0 + a_1 EP_t + a_2 BP_t + a_3 CP_t + a_4 SP_t + a_5 REP_t + a_6 RBP_t + a_7 RCP_t + a_8 RSP_t + a_9 CTEF_t + a_{10} PM_t + e_t$$

where,

- $TR_{t+1}$  = Asset return from period  $t$  to period  $t + 1$ .
- $EP$  = [earnings per share]/[price per share] = earnings–price ratio;
- $BP$  = [book value per share]/[price per share] = book–price ratio;
- $CP$  = [cash flow per share]/[price per share] = cash flow–price ratio;
- $SP$  = [net sales per share]/[price per share] = sales–price ratio;
- $REP$  = [current EP ratio]/[average EP ratio over the past five years];
- $RBP$  = [current BP ratio]/[average BP ratio over the past five years];
- $RCP$  = [current CP ratio]/[average CP ratio over the past five years];
- $RSP$  = [current SP ratio]/[average SP ratio over the past five years];
- $CTEF$  = consensus earnings–per–share I/B/E/S forecast, revisions, and breadth;
- $PM$  = price momentum; and
- $e$  = randomly distributed error term.

The USER model is estimated using a weighted latent root regression (WLRR) analysis on the above returns model to identify variables that are statistically significant at the 10% level, uses the normalized coefficients as weights, and averages the variable weights over the past 12 months. The 12-month smoothing is consistent with the four-quarter smoothing in Guerard and Takano [1991], and Bloch et al. [1993]. We use the values of the coefficients as reported in Guerard et al. [2012a], shown in Exhibit 1 for reference. The USER model evolved from previous models of a similar nature and was recently studied by other researchers in the context of quantitative portfolio construction (see Guerard et al. [2012b]). To the best of our knowledge, an alignment-

oriented discussion of the USER model as presented in this article has never been pursued before. We used the USER model as described in Guerard et al. [2012a] with one small modification, namely, we normalized and win-sorized the factors to eliminate the effect of outliers and also to make them more conducive to regression analysis. Specifically, we centralized the factors to have mean zero, scaled them to have a standard deviation of 1.0, and win-sorized the resulting exposures that were greater than 3.0 in magnitude; the coefficients were modified accordingly to preserve the integrity of the model. We do not expect these modifications to materially alter the characteristics of the USER model. Next, we give some statistics to assess the degree of misalignment between the factors in the USER model and those in the risk models that we used in our experiments.

It is noteworthy that even though there is some overlap between the factors in the USER model and risk factors in US2AxiomaMH, the factors in the respective models are not completely aligned with each other. In order to quantify the degree of alignment, or lack thereof, we borrow a metric from Ceria et al. [2012]. Given an arbitrary factor  $\alpha$  and a risk model exposure matrix  $X$ , consider the following linear regression model that regresses  $\alpha$  against factors in the risk model represented by the matrix  $X$ ,

$$\alpha = Xu + \alpha_{\perp}$$

$\alpha_{\perp}$  denotes the residuals in the above regression model. Let  $\alpha_x = X_u$  denote the portion of  $\alpha$  that is explained by the risk factors  $X$ . There is a subtle connection between the residuals of this regression model and the notion of misalignment, described above. Specifically, if there is no misalignment, then  $\alpha$  is completely explained by the risk factors resulting in vacuous residuals, that is,  $\alpha_{\perp} = 0$ . On the other hand, if the alpha factor is not completely explained by the risk factors, then the coefficient of determination ( $R^2$ ) of the above regression model captures the degree of misalignment between the two set of factors; the higher the  $R^2$ , the smaller the degree of misalignment. In view of this discussion, Ceria et al. [2012]

## EXHIBIT 1

### Coefficients in the USER Model

$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$
0.044	0.038	0.02	0.038	0.089	0.086	0.187	0.122	0.219	0.224

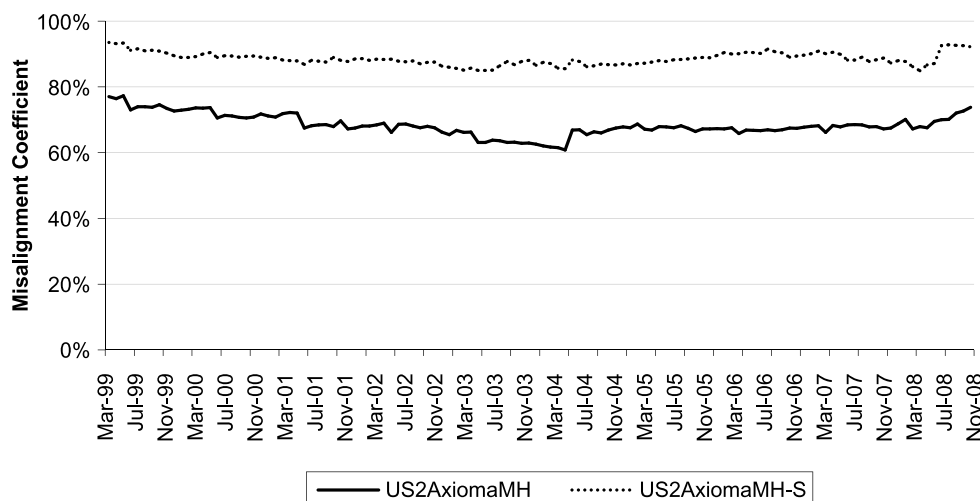
defined the *misalignment coefficient*  $MC(\alpha)$  of alpha to be  $MC(\alpha) = 1 - R^2$ . Borrowing terminology from linear algebra, we refer to  $\alpha_x$  and  $\alpha_\perp$  as the *spanned* and *orthogonal* component of  $\alpha$ , respectively. Unless otherwise stated, we assume that the exposure matrix associated with the US2AxiomaMH risk model is used to define the  $X$  that is used in the regression model described above.

Exhibit 2 shows the time series of the misalignment coefficient of the BP factor in the USER model. While the high MC of BP relative to the statistical risk model is not very surprising, the high MC relative to the

fundamental risk model is indeed intriguing. After all, the “value” factor in US2AxiomaMH risk model is also derived from the B/P ratio. The devil lies in the details; specifically, the USER model and the fundamental risk model apply different kinds of adjustments to the book value before using it to define the B/P factor that results in the demonstrated misalignment. Exhibits 3 and 4 show the time series of MC of the REP and PM factors, while Exhibit 5 provides summarizing MC statistics for all factors in the USER model.

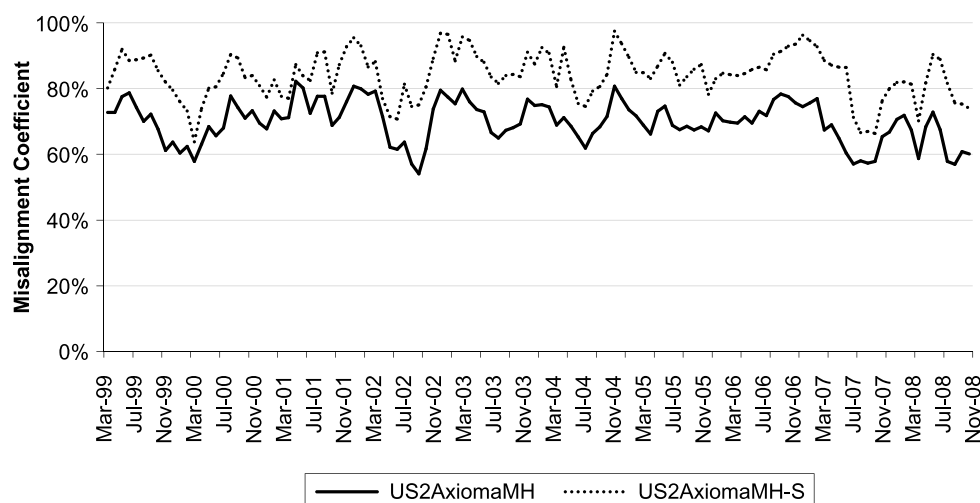
## EXHIBIT 2

### Time Series of Misalignment Coefficient (BP factor)



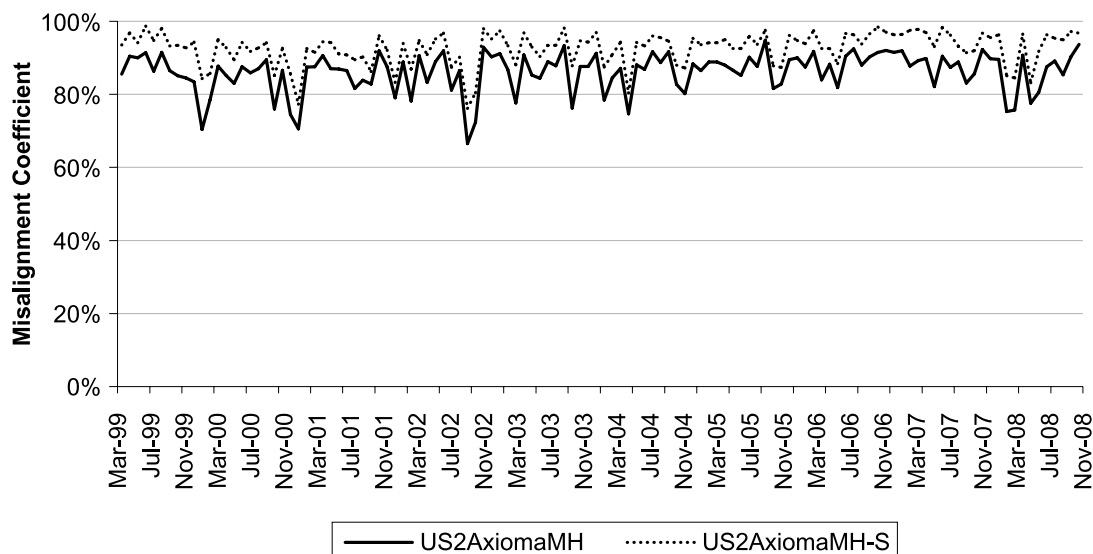
## EXHIBIT 3

### Time Series of Misalignment Coefficient (REP factor)



## EXHIBIT 4

Time Series of Misalignment Coefficient (PM factor)



## EXHIBIT 5

Summary of Alignment Analysis of the Orthogonal Component

Factor	Orthogonal Component				
	Misalignment coefficient (%)	Realized systematic risk (%)	Root-mean-square t-statistic	Percentage of statistically significant periods (90% cf)	Average latent systematic risk exposure (%)
EP	72	66	1.39	25	0.50
BP	68	66	1.77	42	2.13
CP	75	46	1.36	24	0.21
SP	68	43	1.30	21	0.77
REP	70	91	2.13	42	0.37
RBP	65	95	1.88	34	0.19
RCP	65	87	1.82	38	0.23
RSP	89	57	1.58	26	0.23
CTEF	77	70	1.49	27	0.30
PM	86	231	24.30	100	0.32
Alpha	76	131	5.71	98	2.52
Implied-Alpha	1	64	1.31	23	1.50
TO	94	49	1.70	22	0.86
Optimal Portfolio	85	45	1.54	30	1.50

To summarize, there is a significant amount of misalignment between the factors in the USER model and the risk models that we use in this article, which makes their combination ideal for illustrating various facets of FAPs. The section that follows highlights the practical ramifications of this observation in portfolio construction.

### MARKOWITZ EFFICIENCY COMPROMISED

In this section, we describe our experimental setup, discuss computational results, and highlight how FAPs manifest themselves in construction of optimized portfolios. We raise several pertinent questions and set the stage for the following sections to answer them in a systematic fashion.

We used the following strategy in our experiments.

Maximize Expected Return  
s.t

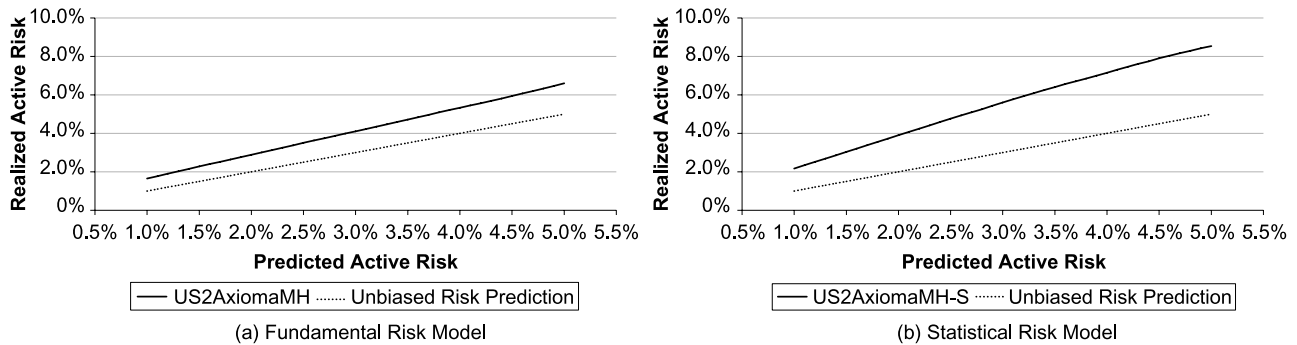
- Fully invested long-only portfolio
- Active GICS sector exposure constraint
- Active GICS industry exposure constraint
- Active asset bounds constraint
- Turnover constraint (two-way; 16%)
- Active Risk constraint ( $\sigma$ )
- Benchmark = Russell 3000

The expected returns were derived using the USER model. We ran monthly backtests based on the above strategy from 1999–2009 for various values of  $\sigma$  chosen from {1.0%, 1.1%, ..., 5.0%}. The backtests were run in two setups that were identical in all respects except for the choice of the risk model; while one setup used Axioma’s fundamental medium-horizon risk model (US2AxiomaMH), the other setup used the

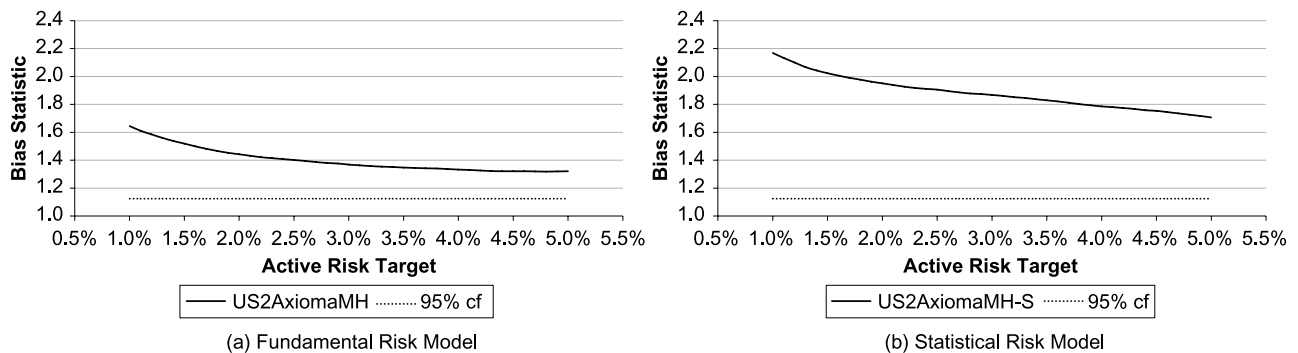
statistical variant (US2AxiomaMH-S) of the same. It is noteworthy that the USER model does not explicitly use risk factors in either of these risk models, the marginal overlap in the definition of some of the factors (B/P, E/P, and so on) notwithstanding. Next we discuss our computational findings.

Exhibit 6 plots the predicted and realized active risk of the portfolios for various risk target levels; for the sake of comparison, we also show a dotted line that corresponds to completely unbiased risk prediction. Note the significant downward bias in risk prediction. Exhibit 7 reports the same information using the concept of the bias statistic. The bias statistic is a statistical metric that is used to measure the accuracy of risk prediction; if the ex ante risk prediction is unbiased, then the bias statistic should be close to 1.0 (see Saxena and Stubbs [2010a] for more details). Clearly, the bias statistics are significantly above the 95% confidence interval, thereby confirming the statistical significance of the downward bias in the risk prediction of optimized portfolios. Finally, Exhibit 8 shows the realized

## EXHIBIT 6 Predicted vs. Realized Active Risk

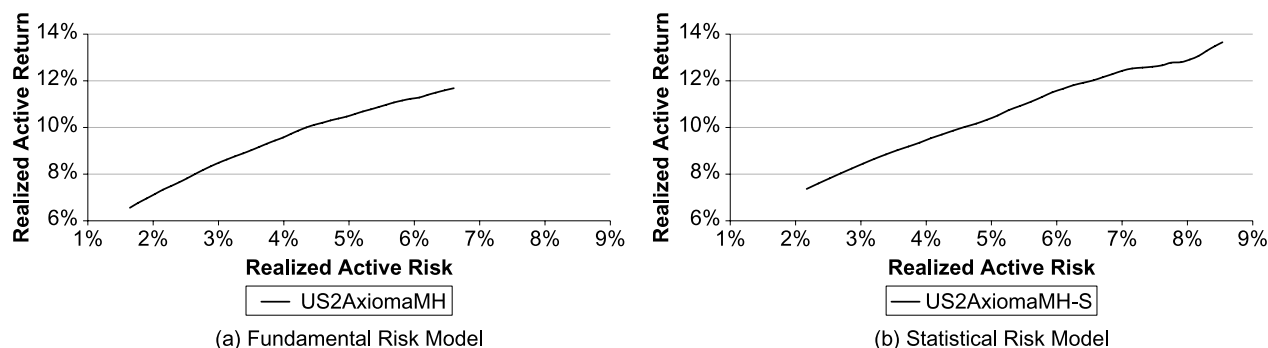


## EXHIBIT 7 Bias Statistics



## EXHIBIT 8

### Realized Active Risk–Return Frontier



active risk–return frontier obtained by using the above strategy. We next focus on optimal portfolios that were generated when a risk target of 3.0% was employed. To simplify the narrative, we limit our discussion to results obtained by using the fundamental risk model; the story is similar with the statistical risk model.

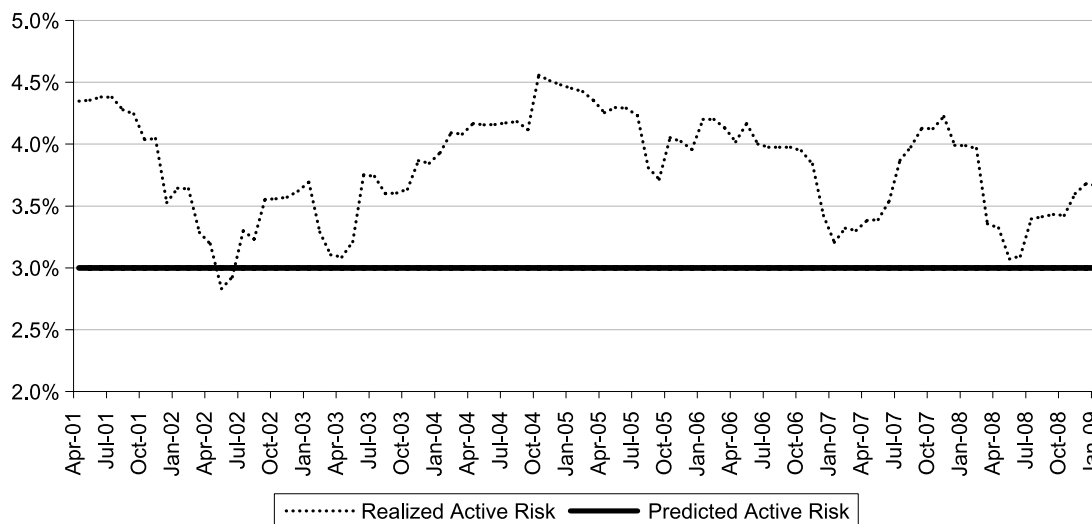
At  $\sigma = 3.0\%$ , the optimal portfolios had realized active risk of 4.11% and active return of 9.72%. The bias statistic for these portfolios was 1.37, which clearly lies outside the 95% confidence interval [0.88, 1.12]. Exhibit 9 further corroborates this phenomenon by showing the time series of realized risk of the optimal portfolios computed using the 24–period realized returns on a rolling–horizon basis; we also show the predicted risk of the portfolios for the sake of comparison. While the degree of under predic-

tion might have varied, the realized risk was consistently above the predicted risk in most of the periods. These portfolios suffer from a serious predicament, namely, even though they were constructed with a risk target of 3.0%, their realized volatility was significantly higher. Among other things, this exposes the portfolio manager to regulatory scrutiny and defeats one of the primary reasons for using a quantitative approach to investing.

Most practitioners are familiar with this phenomenon and use a very simple technique to circumvent it. They calibrate the ex ante risk target level so as to get the desired ex post risk level. For instance, close examination of Exhibit 6 suggests that realized risk of 3.0% can be obtained by using a lower value ( $\sigma = 2.1\%$ ) of the ex ante risk target. As attractive as it may seem and as widely

## EXHIBIT 9

### Time Series of Realized and Predicted Active Risk





used as this approach may be, it has serious shortcomings. For instance, this approach is based on guessing the right ex ante risk target and does not necessarily have a strong economic or financial rationale to back it up. Furthermore, it surgically removes a symptom of FAPs, namely, the downward bias in risk prediction, without necessarily addressing its root cause—excessive and unaccounted exposure to latent systematic risk factors. In other words, adjusting the ex ante risk level is only a symptomatic cure of a more serious and deep, rooted ailment, the FAPs.

To summarize, the primary goal of portfolio optimization is to create a portfolio having an optimal risk-adjusted expected return. If the ex ante risk forecast that is used in pursuit of that goal is itself significantly biased, how can the resulting portfolios be expected to be optimal/efficient? In other words, by virtue of misalignment between the alpha factors, risk factors, and constraints, Markowitz's ultimate dream of accessing efficient portfolios remains unfulfilled. The section that follows performs a post mortem analysis of the optimal portfolios, (implied) alpha, and risk factors to identify what went wrong.

We conclude this section by reporting some of the key style characteristics of optimal portfolios generated using the USER model. Exhibit 10 reports the time series of exposures of the optimal portfolios ( $\sigma = 3\%$ ) to size, value, and momentum factors in US2AxiomaMH.

Evidently, the portfolios had positive exposure to value and momentum factors and negative exposure to the size factor. These characteristics are consistent with results reported in Guerard et al. [2012a], and continue to persist at higher levels of risk targets (see Guerard [2011]).

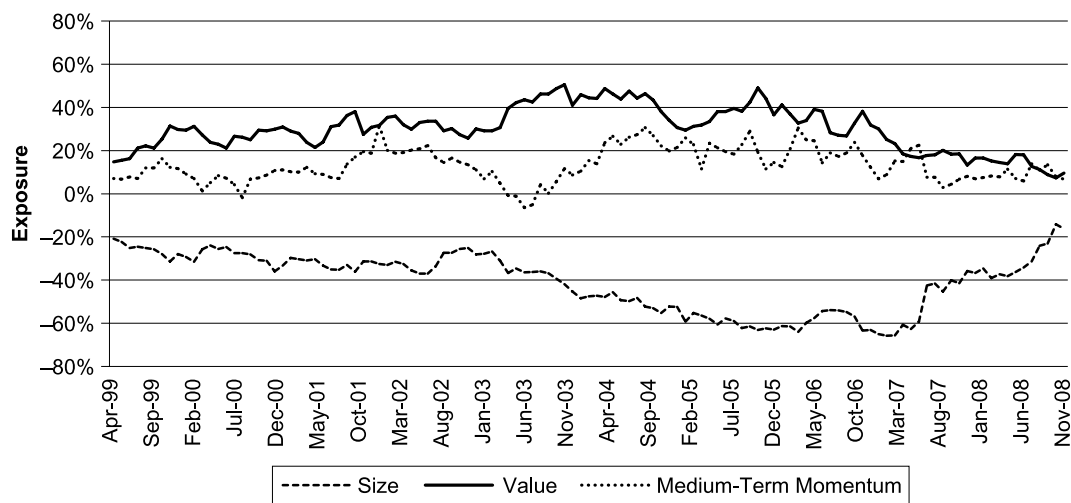
## ALIGNMENT ANALYSIS AND LATENT SYSTEMATIC RISK FACTORS

As previously noted, real-world risk models capture only a subset of all possible systematic risk factors due to practical limitations. Oblivious to this technical subtlety, the optimizer makes an “assumption” that any factor/portfolio that is uncorrelated with the factors in the risk model has no systematic risk, and consequently loads up on the orthogonal component of (implied) alpha. In this section, we give detailed empirical analysis of portfolios derived from the USER model to test the validity of this assumption. We use the setup described in the previous section and report our findings on optimal portfolios corresponding to  $\sigma = 3\%$  derived using the fundamental risk model.

Exhibit 11 reports the time series of the misalignment coefficient of alpha, implied alpha, and the optimal portfolio. Given that the constituent factors of the USER model have high MC (see Exhibits 2,3 and 4), the high MC of alpha is not surprising. The low MC of implied alpha suggests that constraints, specifically, the long-

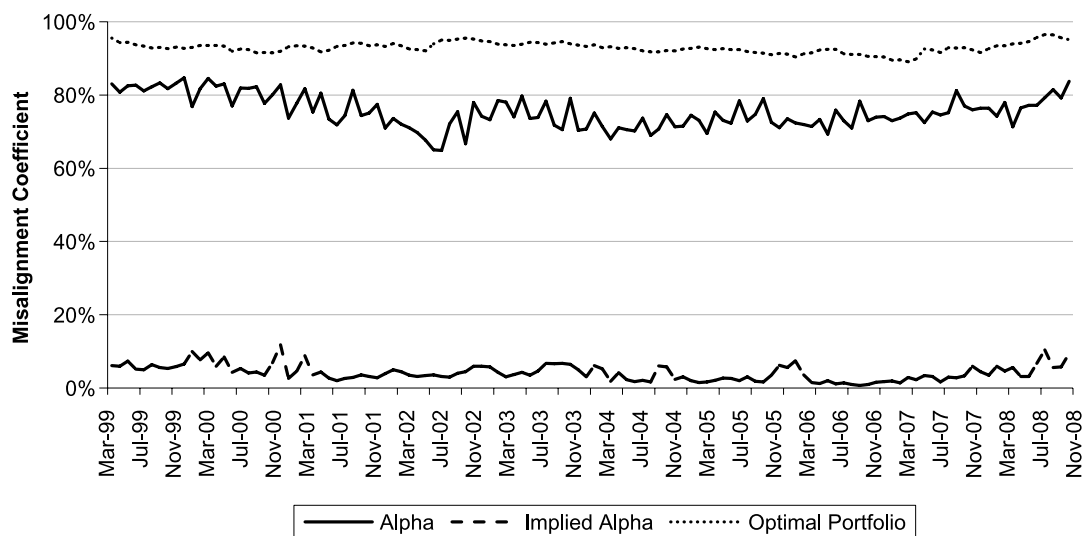
### EXHIBIT 10

Style Exposures of Optimal Portfolios Generated Using the USER Model ( $\sigma = 3\%$ )



## EXHIBIT 11

### Time Series of Misalignment Coefficient



only and turnover constraints, play an important role in determining the optimal portfolio and clipping off the orthogonal component of alpha. Despite the low MC of implied alpha, the optimal portfolio has significantly higher MC, illustrating the degree to which the optimizer favors the orthogonal component relative to the spanned component. Next, we describe a simple experiment to measure the degree of systematic risk in the orthogonal component.

Given an arbitrary factor  $f$ , consider a linear regression model that regresses asset returns against factors in the risk model and the normalized orthogonal component  $\gamma = \frac{f_{\perp}}{|f_{\perp}|}$  of  $f$ . In other words, we augment the suite of risk factors in the original risk model with  $\gamma$ , perform cross-sectional regressions to determine the time series of factor returns that can be attributed to  $\gamma$ , and use the annualized volatility of the resulting factor returns to compute the realized systematic risk in  $f_{\perp}$ . We use a rolling window of 24 periods to determine the time series of realized systematic risk in  $f_{\perp}$ . Additionally, we measure  $t$ -statistics in the cross-sectional regressions to determine the statistical significance of  $f_{\perp}$  as a risk factor. For the sake of brevity, we refer to the mentioned regression model as the *f-augmented regression model*.

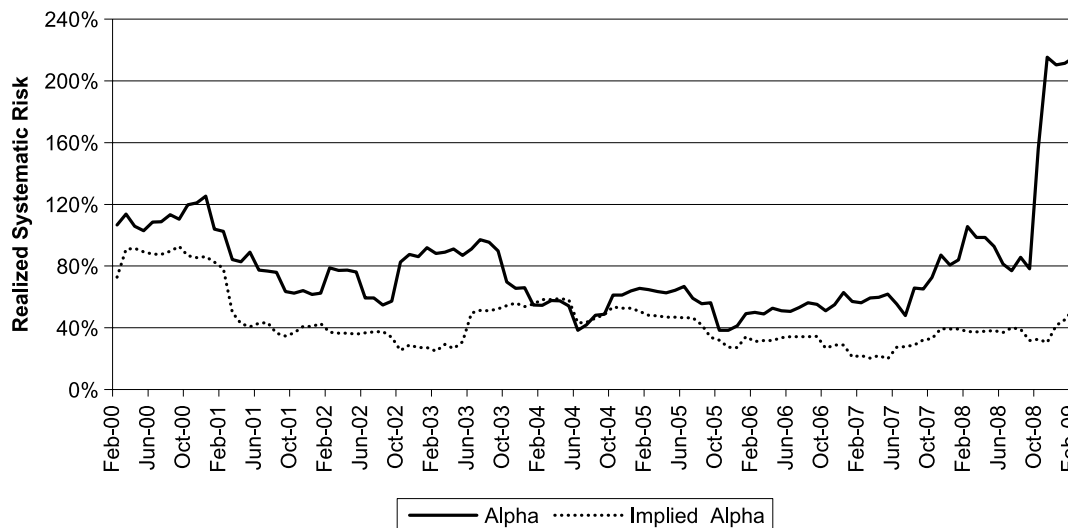
Exhibit 12 reports our key findings. It reports the realized systematic risk, thus in the orthogonal component of alpha and implied alpha. Note that the orthogonal components of both of these entities have significant

systematic risk, thus refuting the argument that being uncorrelated with systematic risk factors in the risk model implies lack of systematic risk exposure. In fact, the realized systematic risk in a median factor of US2AxiomaMH risk model is of the order of 30%–40%,<sup>2</sup> which implies that the orthogonal component of (implied) alpha was better than almost half of the risk factors in the fundamental risk model. Also, note the spike in the realized systematic risk of the orthogonal component of alpha during the 2008–2009 crisis; this suggests emergence of a systematic risk factor, probably counterparty risk or liquidity risk, during the crisis period that was not captured by the fundamental risk model by virtue of its prespecified suite of risk factors. A statistical risk model with more flexibility in choice of risk factors is more suited to capture such transient systematic risk factors. Preliminary investigation suggests that the orthogonal component of the PM factor in the USER model was responsible for the sudden jump in the realized systematic risk of the orthogonal component of alpha; also see Guerard et al. [2012a]. Having demonstrated the existence of latent systematic risk in the orthogonal component, we proceed to identify the sources of the same.

Our starting point is the following characterization of implied alpha ( $\gamma$ ) in terms of the expected return ( $\alpha$ ), shadow prices ( $\pi$ ), and exposure matrix ( $A$ ) associated with the constraints,

## EXHIBIT 12

### Time Series of Realized Systematic Risk of the Orthogonal Component



$$\gamma = \alpha - A^T \pi \quad (1)$$

We refer the reader to Stubbs and Vandembussche [2010] for further discussion of Equation (1) and its relation to the notion of constraint attribution. All entities except the exposure matrix  $A$  in Equation (1) are self-explanatory. In order to understand the construction of  $A$ , consider a strategy that has a factor exposure constraint  $f^T h \geq f_0$ . For instance, the portfolio manager might want to limit exposure to stocks that have limited liquidity, and hence add a factor exposure constraint  $f^T h \geq f_0$  on the liquidity factor  $f$  to derive portfolios with desirable trading characteristics. In this case, the column of  $A$  corresponding to the mentioned factor exposure constraint consists of exposures of the liquidity factor  $f$ . Columns of  $A$  corresponding to other constraints can be derived similarly.

By replacing  $\alpha$  with the USER model, we arrive at a factor structure for implied alpha that includes factors from the USER model (BP, EP, REP, PM, and so on) and hidden factors derived from binding constraints. For each one of the constituent factors, say  $f$ , we computed the time series of following statistics: ( $R$  denotes the reference size of the portfolio,  $h$  denotes the vector of active holdings, and  $\gamma = \frac{f}{|f|}$ ).

1. Exposure of  $h$  to the spanned and orthogonal components of  $f$  given by  $\frac{h^T f_X}{R}$  and  $\frac{h^T f_\perp}{R}$ , respectively.

2.  $t$ -statistics and realized systematic risk of  $\gamma$ , denoted by  $\sigma_\gamma$ , computed using  $f$ -augmented regressions.
3. Latent systematic risk,  $\sigma_y \frac{|h^T \gamma|}{R}$ , of  $h$  that arises by virtue of exposure to  $f_\perp$ .

All of these statistics, except the last one, are self-explanatory. In order to appreciate the computation of latent systematic risk, consider the following argument. The fact that  $\gamma = \frac{f}{|f|}$  has nontrivial systematic risk, despite being uncorrelated with all the risk factors included in the risk model, implies that there exist systematic risk factors beyond those represented in the risk model. As a first order approximation, we can assume that there is exactly one missing systematic risk factor, namely  $\gamma$ , and construct the following augmented risk model to capture it,

$$Q_\gamma = Q + \sigma_\gamma^2 \gamma \gamma^T$$

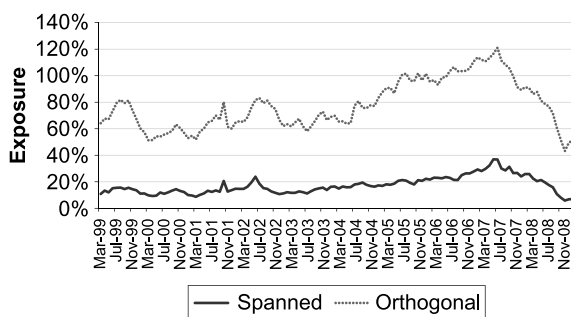
$Q$  denotes the asset–asset covariance matrix implied by the original risk model. In other words, we augment the original suite of risk factors by  $\gamma$  and assume that the factor returns associated with  $\gamma$  are uncorrelated with the factor returns associated with the original set of risk factors. Under these assumptions, the systematic risk of  $h$  that goes undetected during the portfolio construction process is given by  $\sigma_\gamma \frac{|h^T \gamma|}{R}$ . Next, we report our key findings.

Exhibits 13a, 13b, and 13d report the time series of statistics associated with the BP factor. Note that the exposure of the portfolio to the orthogonal component  $BP_{\perp}$  of BP is roughly 3–4 times higher than its exposure to the spanned component  $BP_X$ ; this observation is consistent with the hypothesis that the optimizer perceives no systematic risk in  $BP_{\perp}$ , and hence favors it over  $BP_X$ . This can lead to some serious problems in the implementation of certain kinds of strategies. For instance, if a growth momentum manager employs the USER model in portfolio construction and does not pay attention to the alignment issues, then the resulting portfolios can have excessive exposure to the BP variable, its weight in the USER model notwithstanding. This property of optimized portfolios to inadvertently overrule the weighing of alpha signals used in the expected return model can have serious ramifications for various style-sensitive strategies and partly annuls the effort that goes into the calibration of the alpha model.

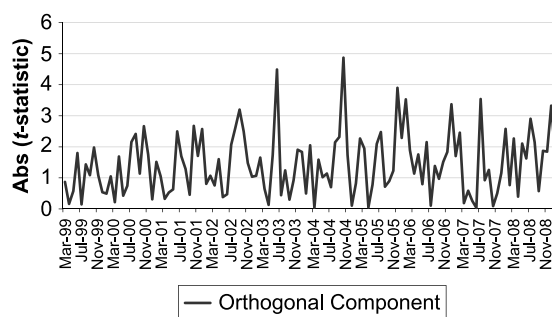
While the optimizer may not regard  $BP_{\perp}$  as a systematic risk factor, the analysis of factor returns derived using the BP-augmented regression model reveals a

completely different picture. Exhibit 13b shows that  $BP_{\perp}$  was statistically significant in 57% of the periods at the 90% confidence level, a surprisingly high value given that a median risk factor in a typical commercial risk model is usually statistically significant in less than 50% of the periods. Furthermore, as shown in Exhibit 13c, the realized systematic risk of  $BP_{\perp}$  was of the order of 40%–60%, which is greater than the systematic risk (30%) of a median factor in US2AxiomaMH. Finally, Exhibit 13d shows that latent systematic risk of the order of 100–200 bps goes undetected during the portfolio construction process due to excessive and inadvertent exposure to  $BP_{\perp}$ . To summarize, not only is  $BP_{\perp}$  a statistically significant risk factor, the optimizer loads up on  $BP_{\perp}$  to the extent that the risk forecast of the optimal portfolio has a downward bias of 100–200 bps. Of course, one should examine these numbers in light of the assumptions that went into the construction of the augmented risk model. Exhibits 14a, 14b, 14c and 14d report the same statistics associated with the REP factor. With individual components of the USER model having statistically significant orthogonal components, it

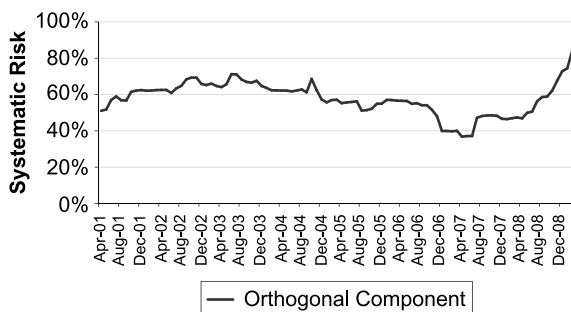
## EXHIBIT 13 Alignment Analysis of the BP Factor



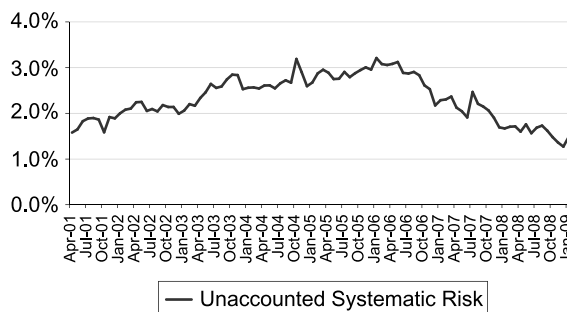
(a) Time Series of Exposures



(b) Time Series of t-statistics (Absolute Values)



(c) Systematic Risk in Orthogonal Component



(d) Unaccounted Systematic Risk of the Optimal Portfolio

is not surprising that the expected returns derived from the USER model betray the same characteristics (see Exhibits 15a, 15b, 15c and 15d). Next, we move our focus to latent systematic risk arising from constraints.

In order to simplify the narrative, we focus exclusively on the turnover constraint. Admittedly the factor in the decomposition (Equation (1)) of implied alpha derived from the turnover constraint (TO factor) is not a conventional risk factor and hence needs special attention. Note that the exposure of asset  $i$  to the TO factor is given by  $|w_i - \bar{w}_i|$ , where  $w_i(\bar{w}_i)$  denotes the final (initial) currency holding of asset  $i$ . Also, note that while the factors of the USER model are normalized to have a mean of 0 and standard deviation of 1, the same cannot be said about the TO factor. Consequently, in order to make the results comparable, we computed the exposure of  $h$  to the spanned and orthogonal component of the TO factor as  $\frac{h^T TO_{\parallel}}{R \|TO_{\parallel}\|}$  and  $\frac{h^T TO_{\perp}}{R \|TO_{\perp}\|}$ , respectively. Exhibits 16a, 16b, 16c and 16d report the key statistics associated with the TO factor. While the systematic nature of the TO factor is not as emphatic as that of the other factors in

the USER model,  $TO_{\perp}$  is still statistically significant, has roughly 20% realized systematic risk, and contributes around 40–80 bps of hidden systematic risk to  $h$ .

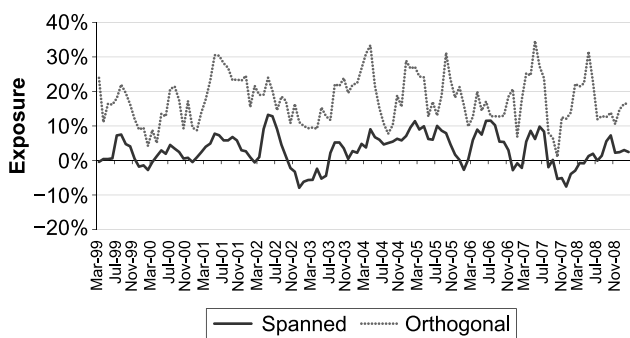
Next, we revisit the time series of predicted and realized risk shown in Exhibit 9, and assess the extent to which our understanding of latent systematic risk factors, as presented above, improves the accuracy of risk forecasts. We use the following expression to compute the adjusted risk estimate that takes into account the systematic risk in the orthogonal component of alpha and the TO factor,

$$\begin{aligned} \left( \begin{array}{c} \text{Adjusted risk} \\ \text{estimate} \end{array} \right)^2 &= \left( \begin{array}{c} \text{Original risk} \\ \text{estimate} \end{array} \right)^2 + \sigma(\alpha_{\perp})^2 \left( \frac{h^T \alpha_{\perp}}{R \|\alpha_{\perp}\|} \right)^2 \\ &+ \sigma(TO_{\perp})^2 \left( \frac{h^T TO_{\perp}}{R \|TO_{\perp}\|} \right)^2 \end{aligned}$$

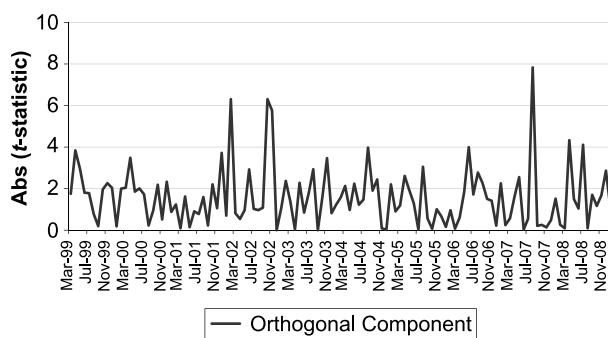
$\sigma(\alpha_{\perp})$  and  $\sigma(TO_{\perp})$  denote the realized systematic risk in the orthogonal component of  $\alpha$  and the TO factor,

## EXHIBIT 14

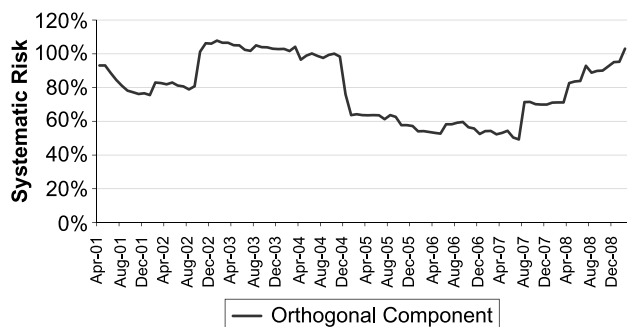
### Alignment Analysis of the REP Factor



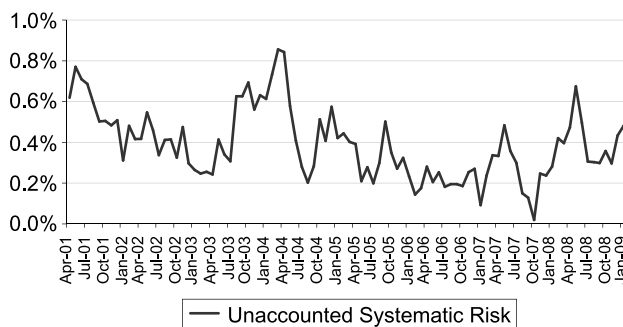
(a) Time Series of Exposures



(b) Time Series of t-statistics (Absolute Values)

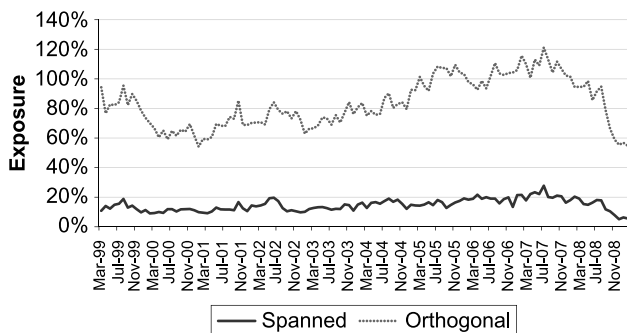


(c) Systematic Risk in Orthogonal Component

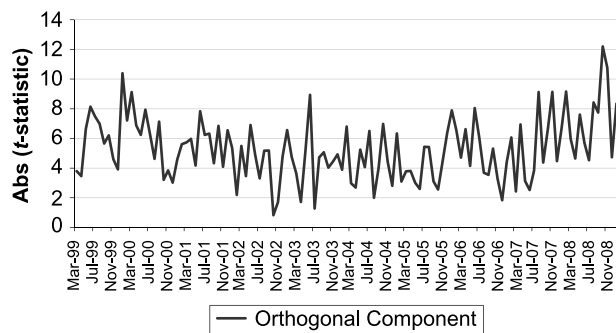


(d) Unaccounted Systematic Risk of the Optimal Portfolio

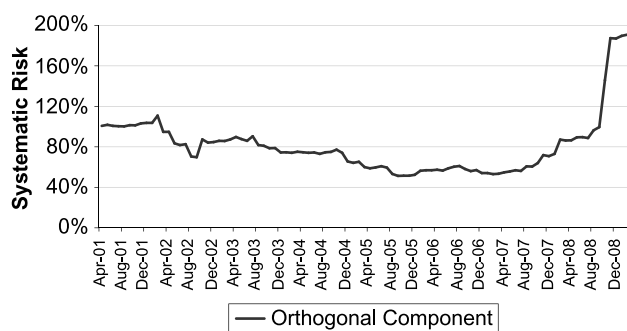
## EXHIBIT 15 Alignment Analysis of Alpha



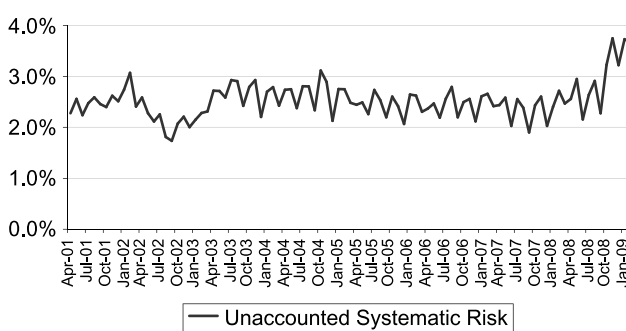
(a) Time Series of Exposures



(b) Time Series of *t*-statistics (Absolute Values)



(c) Systematic Risk in Orthogonal Component



(d) Unaccounted Systematic Risk of the Optimal Portfolio

respectively, determined using augmented regression models. Note that in addition to the assumptions of the augmented risk model described earlier, the above computation also assumes that the systematic risk in  $\alpha_{\perp}$  and  $TO_{\perp}$  arises due to two different mutually uncorrelated systematic risk factors that are missing from the risk model. A refinement of the above computation that circumvents these assumptions entails the generation of custom risk models and goes beyond the scope of this article.

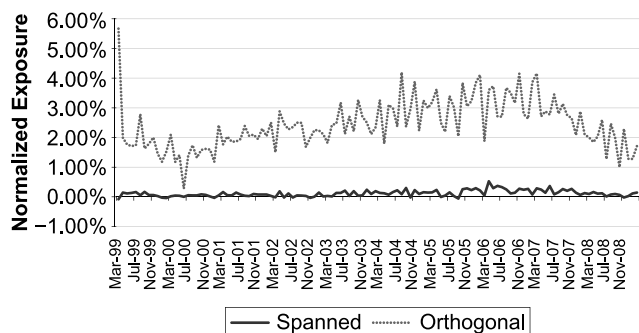
Exhibit 17 reports the time series of adjusted risk estimates; for the sake of comparison, we also reproduce the time series of realized and predicted risk reported in Exhibit 9. Note that the adjusted estimate tracks the realized risk better than the predicted estimate obtained by using the original risk model; the bias statistic using the adjusted risk estimate is 0.93, which lies within the 95% confidence interval [0.88, 1.12] and suggests that accounting for the latent systematic risk factors removes the downward bias in risk prediction to a large extent.

Exhibit 5 provides a summary of results presented in this section. It is interesting to note that even though the orthogonal component of the PM factor was highly

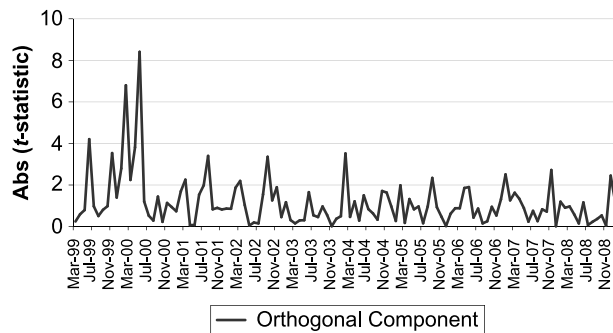
statistically significant (RMS *t*-statistics = 23) and had considerable systematic risk (231%), its contribution to the latent systematic risk in the optimal portfolio was mediocre at best. This is consistent with the observation that volatility of a latent systematic risk factor, by itself, is an inadequate measure to determine the extent of latent systematic risk. Instead, it should be used in conjunction with the exposure of the optimal portfolio to the respective factor to determine the amount of hidden systematic risk. Compared on the basis of average latent systematic risk exposure, BP, TO (turnover), and SP factors appear to be the top three factors. The appearance of the TO factor in this list is rather surprising. While most quantitative managers acknowledge that misalignment between the alpha and risk factors can lead to unintended bets, it is quite an intriguing fact that turnover limitations can also introduce latent systematic risk in the portfolio. In other words, FAPs arising due to misalignment between the risk factors and constraints can be equally important as those arising due to misalignment between the risk factors and alpha factors.

# EXHIBIT 16

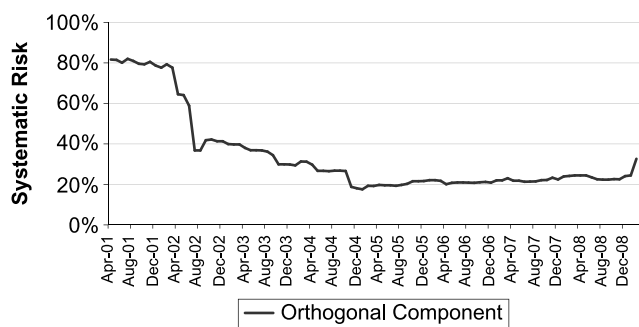
## Alignment Analysis of the TO (turnover) Factor



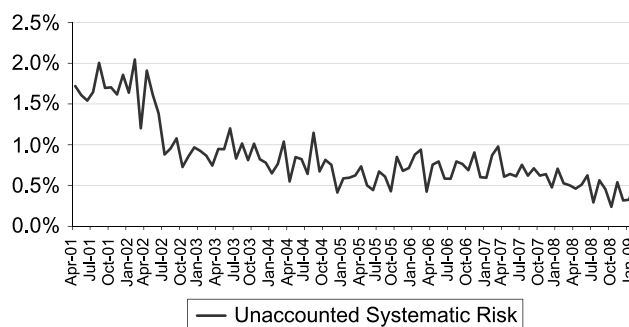
(a) Time Series of (normalized) Exposures



(b) Time Series of  $t$ -statistics (Absolute Values)



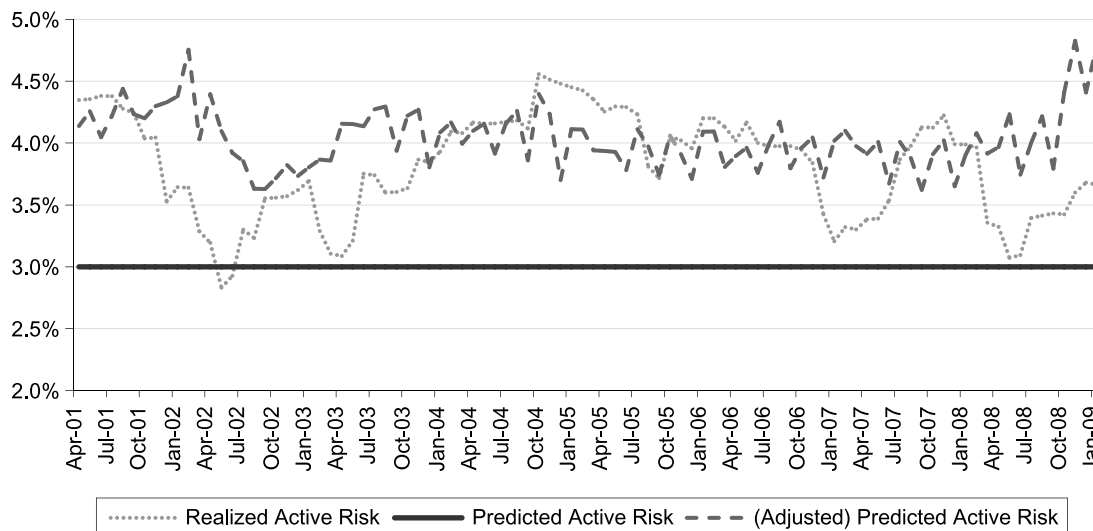
(c) Systematic Risk in Orthogonal Component



(d) Unaccounted Systematic Risk of the Optimal Portfolio

# EXHIBIT 17

## Time Series of Adjusted Risk Estimates



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Up to this point, we have highlighted the malady of FAPs and trace their sources. Now, we leverage the insight thus obtained to propose an effective remedy to FAPs and illustrate the efficacy of the proposed methodology.

## MARKOWITZ EFFICIENCY RESTORED

The key idea in devising a solution to FAPs is the observation that the optimizer unknowingly takes exposure to certain systematic risk factors that are missing from the risk model. While it is difficult to characterize these latent risk factors, we do know that they are present in the orthogonal component of implied alpha ( $\gamma$ ) and owe their origins to either the components of alpha (BP, REP, PM, and so on) or the binding constraints (turnover constraint, long-only constraint, and so on.) In fact, Saxena and Stubbs [2010b] showed that only those systematic risk factors that have significant overlap with  $\gamma_{\perp}$  have a bearing on FAPs. Consequently, a natural solution to FAPs is to use an augmented risk model,  $Q_y = Q + \nu\gamma\gamma^T$ , in portfolio construction where  $\gamma = \frac{1}{\|\gamma_{\perp}\|}\gamma_{\perp}$ . The augmented risk model penalizes the exposure of the portfolio to  $\gamma_{\perp}$  and thus avoids unintended bets on missing risk factors.

There is only one implementation challenge in using  $Q_y$ , namely, that  $\gamma$  is a dynamic entity that cannot be determined a priori. Interestingly, it can be shown that under assumptions of homoscedasticity,  $\frac{1}{\|\gamma_{\perp}\|}\gamma_{\perp} = \frac{1}{\|h_{\perp}\|}h_{\perp}$ , where  $h$  denotes the optimal holdings. Using this result, the variance of the portfolio using  $Q_y$  can be formulated as:

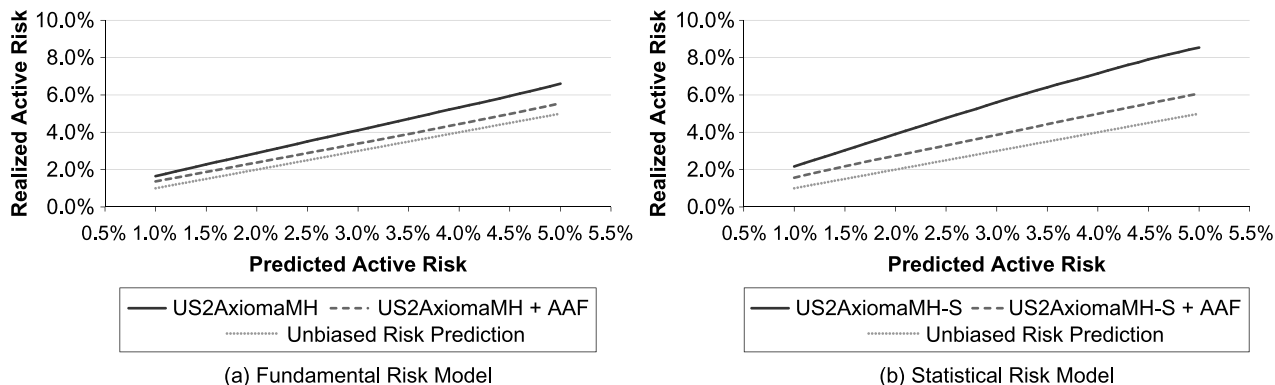
$$h^T Q_y h = h^T Q h + \nu \|h_{\perp}\|^2$$

which in turn can be formulated as a convex optimization problem (see Saxena and Stubbs [2010a] for details). The factor  $\frac{1}{\|h_{\perp}\|}h_{\perp}$  is called the alpha alignment factor (AAF), and the resulting approach to portfolio construction is referred to as the AAF approach. Next, we discuss the results obtained by applying the AAF approach to the USER model. We used the realized systematic risk of  $h_{\perp}$  reported in Exhibit 5 to determine the value of  $\nu$ . In practice,  $\nu$  can be calibrated using the historical time series of factor returns attributable to  $h_{\perp}$ .

Exhibit 18 shows the plot of predicted versus realized active risk; for the sake of comparison, we also reproduce the results obtained without the AAF approach. As evident from the figure, using the AAF approach significantly reduces the bias in risk prediction. Exhibit 19 shows the same plot using the concept of bias statistic. Note that risk forecasts obtained by using the AAF approach are unbiased at the 95% confidence level when the fundamental risk model is employed; the bias is significantly reduced when the AAF approach is used in conjunction with the statistical risk model. Among other things, this implies that using the AAF approach eliminates the need to use a lower risk target to accommodate for the bias in risk prediction. Furthermore, unlike the alternative ad hoc approach based on “guessing” the right ex ante risk level, the AAF approach has a strong theoretical foundation (see Saxena and Stubbs [2010b]) and is based on an empirically verifiable hypothesis concerning the existence of latent systematic risk factors (see previous section).

## EXHIBIT 18

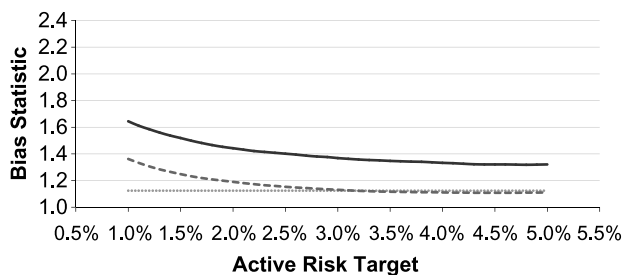
### Predicted vs. Realized Active Risk Using AAF





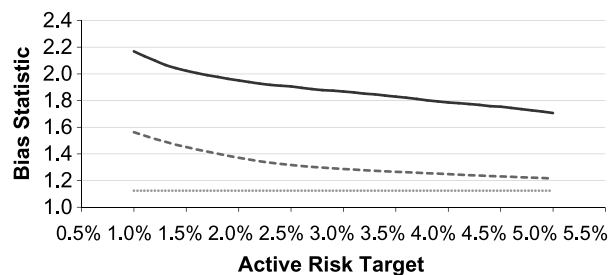
## EXHIBIT 19

### Bias Statistics Using AAF



— US2AxiomaMH ..... 95% cf --- US2AxiomaMH + AAF

(a) Fundamental Risk Model



— US2AxiomaMH-S ..... 95% cf --- US2AxiomaMH-S + AAF

(b) Statistical Risk Model

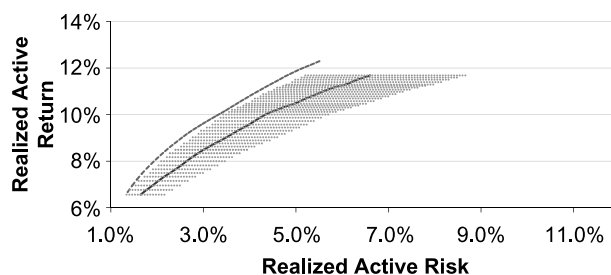
Finally, Exhibit 20 reports the realized risk–return frontier. Note that using the AAF approach not only improves the accuracy of risk prediction but also enhances ex post risk-adjusted performance. In other words, unlike other solutions to risk underestimation problems that merely move the portfolio on the efficient frontier, the AAF approach pushes the frontier upward, allowing the portfolio manager to access portfolios that lie above the traditional risk–return frontier. In Exhibit 20 we also show the 95% confidence interval around the original frontier to highlight that the improvements obtained by using the AAF approach are statistically significant. How can an approach that is designed exclusively to improve the accuracy of risk prediction and that does not use a better expected return model yield statistically significant improvements in ex post performance?

The answer to this question lies in the pivotal role that risk models play in the construction of optimized

portfolios. The influence of risk models is not simply limited to obtaining the ex ante risk forecasts. Instead, they materially affect the composition of optimal holdings, budget, and risk allocation across various securities, turnover utilization, and primary characteristics of interest, such as information ratio, Sharpe ratio, transfer coefficient, and so on. Naturally, if there are systematic biases in the optimal portfolio that are not captured by the risk model, all of these mentioned characteristics get affected, resulting in inefficient risk and budget allocation. By recognizing and correcting for the existence of unaccounted systematic risk factors, the AAF approach makes holistic improvements to the process of portfolio construction, which results in not only better risk forecasts but also improved ex post performance, thereby restoring Markowitz’s MVO efficiency. In order to substantiate this argument, we conducted the following experiment.

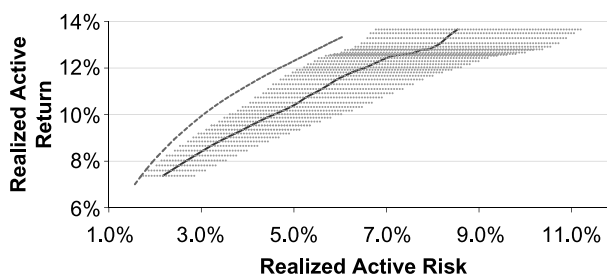
## EXHIBIT 20

### Realized Active Risk–Return Frontier Using AAF



— US2AxiomaMH ..... US2AxiomaMH + AAF  
..... 95% cf

(a) Fundamental Risk Model

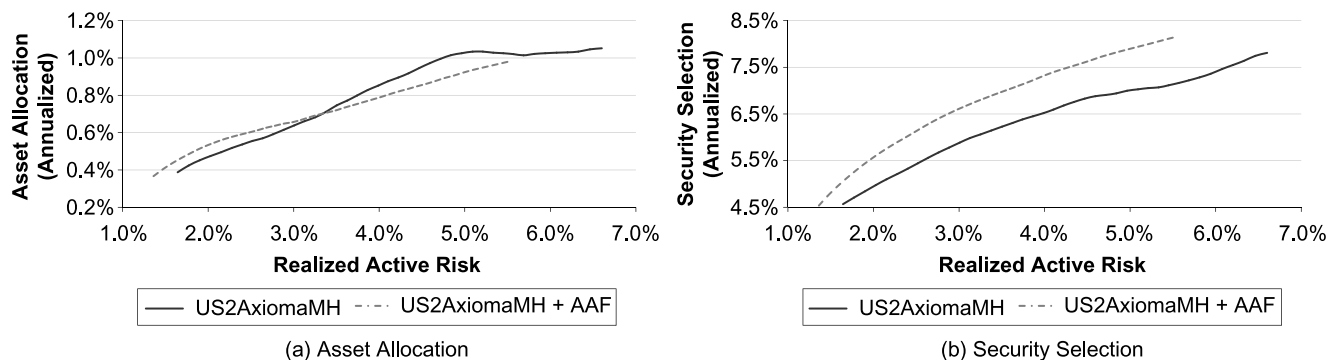


— US2AxiomaMH-S ..... US2AxiomaMH-S + AAF  
..... 95% cf

(b) Statistical Risk Model

## EXHIBIT 21

### Returns-Based Attribution Analysis for Portfolios Generated Using the Fundamental Risk Model (US2AxiomaMH)



We generated the time series of optimal holdings obtained by using the US2AxiomaMH risk model both with and without the AAF methodology for various values of the risk target  $\sigma \in \{1\%, 1.1\%, \dots, 5\%\}$ . Subsequently, we performed returns-based attribution analysis on the resulting holdings using Axioma's performance attribution toolkit; we used the GICS industry classification scheme as adapted in US2AxiomaMH for this purpose. Exhibit 21 reports the key findings. Three remarks are in order.

First, note that most of the active returns can be attributed to security selection, with only a marginal amount derived using asset allocation. This observation attests to the ability of the USER model to identify over- and undervalued securities in each industry. These security selection statistics are consistent with the results reported in Guerard et al. [2012a] and are known to be statistically significant (Guerard [2011]). Second, using the AAF has insignificant impact on the component of active returns that can be attributed to asset allocation. Third, portfolios generated using the AAF methodology had roughly 60–80 bps better security selection than those generated using the plain-vanilla risk model. These observations suggest that factor alignment problems inhibit the ability of the optimizer to fully leverage the potential of the USER model apropos security selection; by directly addressing the misalignment problem, the AAF facilitates the optimizer to transfer a greater amount of novelty and information in the USER model to the resulting optimal portfolios, thereby unlocking the latent security selection potential of the expected return model that a misaligned risk model is unable to capitalize on.

We want to emphasize that the improvements illustrated in Exhibit 20 have a strong theoretical foundation. In fact, it can be shown that under certain assumptions, the AAF approach is guaranteed to yield not only unbiased risk forecasts, but also ex post performance improvements of the kind illustrated in Exhibit 20. We refer the reader to Saxena and Stubbs [2010b] for further details.

## CONCLUSION

In this article, we set out to illustrate FAP—sources, effects, analyses, and solutions—on portfolios constructed using expected returns derived from the USER model. We highlighted that the factors in the USER model have high misalignment coefficients mirroring the situation faced by most quantitative managers. As expected, optimal portfolios generated using the USER model without adjusting for alignment issues betray the typical symptoms of FAPs—downward bias in risk prediction, high MC of implied alpha and optimal holdings, statistically significant orthogonal components, and most importantly, exposure to latent systematic risk factors. Furthermore, we were able to trace the unaccounted systematic risk to constituent factors of the USER model and the turnover constraint. Assembling pieces of information garnered during this analysis and using it to update the ex ante risk forecasts eliminated the bias in predicted risk of optimized portfolios. Finally, we leveraged our understanding of latent systematic risk factors to modify the portfolio construction process so as to generate portfolios that are resilient to FAPs. We used

the AAF approach to pursue this goal, and demonstrated that using the mentioned approach improves not only the accuracy of risk forecasts but also the ex post performance in a statistically significant manner.

We would like to emphasize the role of augmented risk models in guiding our analysis. By virtue of their analytical simplicity, augmented risk models naturally lend themselves to detailed theoretical analysis. Of course, one of the key assumptions—namely, that the missing factors are uncorrelated with the existing risk factors—that underlies augmented risk models need not hold true in practice. We believe that surmounting this limitation might hold clues to further improving the results presented in this article. Using customized risk models obtained by recalibrating the original risk model by explicitly incorporating the alpha factors in the risk model is a promising research direction to accomplish this goal.

## ENDNOTES

The first author would like to thank John Guerard for providing access to the USER model and for useful discussions that helped improve the quality of this article. Thanks are also due to Vishnu Anand for his help in setting up the backtests using the USER and Axioma's risk models.

<sup>1</sup>Following Ceria et al. [2012], we use the terms expected return and alpha synonymously.

<sup>2</sup>For the sake of comparability, we scaled the factors in US2AxiomaMH to have  $l_2$  norm of 1.0 before computing the mentioned realized systematic risk; the orthogonal component of (implied) alpha was scaled similarly.

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