Efficient global portfolios: Big data and investment universes

J. B. Guerard, Jr.
S. T. Rachev
B. P. Shao

In this analysis of the risk and return of stocks in the United States and global markets, we apply several portfolio construction and optimization techniques to U.S. and global stock universes. We find that (1) mean-variance techniques continue to produce portfolios capable of generating excess returns above transaction costs and statistically significant asset selection, (2) optimization techniques minimizing expected tail loss are statistically significant in portfolio construction, and (3) global markets offer the potential for greater returns relative to risk than domestic markets. In this experiment, mean-variance, enhanced-index-tracking techniques, and mean-expected tail-loss methodologies are examined. Global equity data and the vast quantity (and quality) of the data relative to U.S. equity modeling have been discussed in the literature. We estimate expected return models in the U.S. and global equity markets using a given stock-selection model and generate statistically significant active returns from various portfolio construction techniques.

Introduction

In this study, we apply several portfolio construction and optimization techniques to U.S. and global stock universes. Global equity data and the vast quantity (and quality) of the data relative to U.S. equity modeling have been discussed in the literature. We estimate expected return models in the U.S. and global equity markets using a given stock-selection model and generate statistically significant active returns from various portfolio construction techniques.

This paper is organized as follows. In the section "Constructing mean-variance efficient portfolios," we introduce the reader to the risk and return tradeoff analysis. In the section "Extensions to the mean-variance optimization model," we discuss the Markowitz enhanced-index-tracking (EIT) portfolio construction model in which security weights are an absolute deviation from the security weight in the index. We refer to the absolute deviation from the benchmark weight enhanced-index portfolio construction weight as the equal active weighting (EAW) portfolio construction model. In the section "Arbitrage pricing theory multi-factor models in business," we discuss the theory of multi-factor risk models. In the section "A general stock-selection model

for U.S. and global equity markets," we discuss our data sources and the construction and estimation of a general stock-selection model for U.S. and global securities. In the section "Extension to mean-ETL optimization model," we discuss mean-expected tail-loss (ETL) optimization. In the section "Portfolio simulation results with the U.S. Expected Returns (USER) and Global Expected Returns (GLER) models," we discuss portfolio construction and simulation, and present the empirical results. In the conclusion, we offer conclusions and a summary.

We report that (1) mean-variance (MV) techniques continue to produce portfolios capable of generating excess returns above transactions costs and statistically significant asset selection, (2) optimization techniques minimizing expected tail loss are statistically significant in portfolio construction, and (3) global markets offer the potential for greater returns relative to risk than domestic markets. In this experiment, MV, EIT techniques, and mean-ETL methodologies are examined.

H. M. Markowitz developed a portfolio construction model to achieve the maximum return for a given level of risk or the minimum risk for a given level of return [1–4]. It has long been noted by B. Solnik [5, 6] that investors should diversify internationally rather than domestically, and

the number of international securities is much larger than the securities of the U.S. market, aiding diversification. To better extend the portfolio construction methodology and techniques used for the U.S. market to the international market, we briefly review the applied U.S. equity investment research presented in Guerard et al. [7] and Tsuchida et al. [8]. Creating portfolios using statistics-based risk models for the U.S. and global markets, we test whether global markets offer the potential for greater returns relative to risk than domestic markets. We test MV, EIT techniques, and mean-ETL methodologies on the global assets.

In constructing efficient portfolios, the security weights are the primary decision variables to be solved. Second, we test whether the MV optimization technique using the portfolio variance as the relevant risk measure dominates the risk/return tradeoff curve using a variation of the MV optimization model that emphasizes systematic (or market) risk, the MV Tracking Error at Risk (MVTaR) optimization, and mean-ETL portfolio optimization. A statistics-based Principal Components Analysis model is used to estimate and monitor portfolio risk.

A measure of the tradeoff between the portfolio expected return and risk (as measured by the portfolio standard deviation) is typically denoted by the Greek letter lambda (λ). Generally, the higher the lambda, the higher is the ratio of portfolio expected return to portfolio standard deviation. We assume that the portfolio manager seeks to maximize the portfolio geometric mean (GM) and Sharpe ratio as put forth in Latane [9] and Markowitz [3, 10].

Summary and findings
We report that the Markowitz MV optimization technique, the EIT optimization technique, and the MVTaR optimization technique are appropriate tools for portfolio construction for the U.S. Expected Returns (USER) and Global Expected Returns (GLER) data. Global portfolios dominate domestic portfolios with regard to the return-to-risk statistics for the 1999 to 2011 period. The Markowitz approach to portfolio construction and management is 60 years old and remains an integral tool of investment research.

Constructing mean-variance efficient portfolios
Portfolio construction and management, as formulated in Markowitz, seeks to identify the efficient frontier, the point at which the portfolio return is maximized for a given level of risk, or equivalently, portfolio risk is minimized for a given level of portfolio return. The portfolio expected return, denoted by $E(R_P)$, is calculated from the sum of the security weights multiplied by their respective expected returns:

$$E(R_P) = \sum_{i=1}^{N} w_i E(R_i), \tag{1}$$

where N is the number of candidate securities, w_i is the weight for security i, and $\sum_{i=1}^{N} w_i = 1$ indicating that the portfolio is fully invested. $E(R_i)$ is the expected return for security i. The portfolio standard deviation σ_P is the square root of portfolio variance σ_P^2, which is the sum of the weighted securities covariance:

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}, \tag{2}$$

where σ_{ij} is the covariance of returns for security i and security j.

The Markowitz framework measures risk as the portfolio standard deviation, a measure of dispersion or total risk. One seeks to minimize risk, as measured by the covariance matrix in the Markowitz framework, holding constant expected returns. The decision variables estimated in the Markowitz model are the security weights. The Markowitz model minimized the total risk, or variance, of the portfolio. Investors are compensated for bearing total risk.

Extensions to the mean-variance optimization model
We introduce two extensions to the MV approach: an EIT optimization technique and a tracking error at risk optimization technique. Markowitz [4] rewrites the general portfolio construction model variance, V, to be minimized as

$$V = (\mathbf{X} - \mathbf{W})^{\mathrm{T}} \mathbf{C} (\mathbf{X} - \mathbf{W}), \tag{3}$$

where $\mathbf{W}^{\mathrm{T}} = (W_1, \ldots, W_n)^{\mathrm{T}}$ is the vector of weights of an index of returns, $\mathbf{X}$ are the portfolio weights, and $\mathbf{C}$ are the covariance of returns. $(\cdot)^{\mathrm{T}}$ is the transpose. Guerard et al. [11] reported the efficiency of the EIT procedure is minimizing realized tracking errors.

One creates portfolios by allowing portfolio weights to differ from index weights by plus or minus 1% up to 5%, each portfolio denoted by EAW followed by a number indicating the percent. Obviously, one can use an infinite set of EAW variations. Guerard et al. [12] employed MV and EIT optimization techniques to test whether EAW strategies of 1%, 2%, 3%, 4%, and 5% (weight deviations from the index, or benchmark, weights) outperform MV strategies using 4% and 7% maximum security weights. Guerard et al. [12] also reported that MV portfolios produced higher information ratios and Sharpe ratios than EAW portfolios with weights less than EAW4 (portfolio weights are allowed to differ from index weights by ±4%). Thus, the traditional Markowitz MV is (still) quite relevant in the world of business.

Arbitrage pricing theory multi-factor models in business
Earlier, we introduced the reader to the Markowitz model in which investors are compensated for bearing the total risk

of the portfolio. Implicit in the development of the Capital Asset Pricing Model by Sharpe [13], Lintner [14], and Mossin [15] is that investors are compensated for bearing systematic or market risk, not total risk. Systematic risk is measured by the beta of a stock. Beta is the slope of the market model in which the stock return is regressed as a function of the market return. Sharpe [16] proposed what he refers to as the diagonal model to simplify the computations in constructing portfolios. An investor is not compensated for bearing risk that may be diversified away from the portfolio, but rather portfolio risk as measured by factor exposures.

Multi-factor risk models evolved in the works of Rosenberg [17], Ross [18], and Ross and Roll [19]. The fundamentally based domestic Barra risk model was developed in Rosenberg, Rosenberg, and Marathe [20] and thoroughly discussed in Rudd and Clasing [21] and Grinold and Kahn [22]. Barra attribution is used in this analysis to access stock-selection statistical significance. The Barra model remains the industry standard for risk model. The total excess return for a multiple-factor model in the Rosenberg methodology for security j may be written as follows:

$$R_j = \sum_{k=1}^{K} \beta_{jk} \widetilde{f}_k + \widetilde{e}_j. \tag{4}$$

The nonfactor, or asset-specific return $\widetilde{e}_j$ on security j, is the residual risk of the security after removing the estimated impacts of the K factors. The term $\widetilde{f}_k$ is the rate of return on factor k, and β_{jk} is factor beta of security j on factor k. An extensive review of factor risk models can be found in Connor and Korajczyk [23].

Guerard [24] demonstrated the effectiveness of the Blin and Bender arbitrage pricing theory (APT) and Sungard APT systems in portfolio construction and management. The determination of security weights, the ws, in a portfolio are the primary calculation of the Markowitz portfolio management approach. The security weight is the proportion of the portfolio value invested in the individual j security.

The portfolio weight of security j is calculated as:

$$w_{(\mathrm{P})j} = \frac{MV_j}{MV_\mathrm{P}}, \tag{5}$$

where MV_j is the market value of security j, and MV_P is the market value total portfolio.

The active weight of the security j, $w_{(\mathrm{a})j}$ is calculated by subtracting the security weight in the (index) benchmark b, $w_{(\mathrm{b})j}$, from the security weight in the portfolio:

$$w_{(\mathrm{a})j} = w_{(\mathrm{P})j} - w_{(\mathrm{b})j}. \tag{6}$$

Blin and Bender created their Advanced Portfolio Technologies company, and its *Analytics Guide* [25, 26],

which built upon the mathematical foundations of their APT system, published in Blin et al. [27]. Our review draws upon the *Advanced Portfolio Technologies Analytics Guide*. Volatility can be decomposed into independent variance components, systematic risk, and specific risk:

$$\sigma_\mathrm{P}^2 = \sigma_{\beta\mathrm{P}}^2 + \sigma_\mathrm{SP}^2, \tag{7}$$

where σ_P^2 is the total portfolio variance, $\sigma_{\beta\mathrm{P}}^2$ is the systematic portfolio volatility, and σ_SP^2 is the specific portfolio volatility.

Tracking error is a measure of volatility applied to the active return of funds (portfolio) benchmark with respect to an index. Portfolio tracking error is defined as:

$$\begin{aligned}\sigma_\mathrm{te} &= \sqrt{\mathrm{Var}(\mathbf{r}_\mathrm{P} - \mathbf{r}_\mathrm{b})} \\ &= \sqrt{E[(\mathbf{r}_\mathrm{P} - \mathbf{r}_\mathrm{b}) - E(\mathbf{r}_\mathrm{P} - \mathbf{r}_\mathrm{b})]^2},\end{aligned} \tag{8}$$

where σ_te is the square root of the variance of annualized tracking error, and $\mathbf{r}_\mathrm{P}$ and $\mathbf{r}_\mathrm{b}$ are the actual (annual) portfolio return and benchmark return, respectively. Systematic tracking error of a portfolio is a forecast of the portfolio active annual returns as a function of the securities returns associated with APT risk (factor) model components. The difference in APT portfolio returns versus a benchmark return at time t can be written as:

$$d_{\mathrm{P,b}}^{(t)} = \left(\sum_{i=1}^{n_\mathrm{P}} \mathbf{w}_{(\mathrm{P})i} \mathbf{r}_{ti} - \sum_{j=1}^{n_b} \mathbf{w}_{(\mathrm{b})j} \mathbf{r}_{tj} \right), \tag{9}$$

where n_P is the number of securities in the portfolio, and n_b is the number of securities in the portfolio benchmark. $\mathbf{r}_{ti}$ and $\mathbf{r}_{tj}$ are the returns of security i and j at time t, respectively.

Although portfolios often contain stocks not in the benchmark, we can make $n_\mathrm{P} = n_\mathrm{b} = m$ by inserting zeros in the weights when appropriate. Let us define column vectors $\mathbf{w}_\mathrm{P}$ and $\mathbf{w}_\mathrm{b}$ for given portfolio and benchmark portfolios, respectively. Row vector $\mathbf{r}_t$ denotes the security returns at time t. Then, difference in APT portfolio returns can be rewritten as:

$$d_{\mathrm{P,b}}^{(t)} = \mathbf{r}_t \cdot (\mathbf{w}_\mathrm{P} - \mathbf{w}_\mathrm{b}). \tag{10}$$

Blin and Bender mimic the APT model in the *Analytics Guide*:

$$\begin{aligned}\sigma_{\mathrm{P,b}}^2 &= \mathrm{Var}(\mathbf{d}_{\mathrm{P,b}}) \\ &= (\mathbf{w}_\mathrm{P} - \mathbf{w}_\mathrm{b})^\mathrm{T}(\mathbf{B}^\mathrm{T}\mathbf{B} + \Sigma)(\mathbf{w}_\mathrm{P} - \mathbf{w}_\mathrm{b}),\end{aligned} \tag{11}$$

where $\mathbf{B}$ is the $k \times m$ matrix of factor loading, and $\Sigma = \mathbf{e}'\mathbf{e}$ is the $m \times m$ diagonal matrix of the specific risk loading. $\mathbf{d}_{\mathrm{P,b}} = [d_{\mathrm{P,b}}^{(1)}, \ldots, d_{\mathrm{P,b}}^{(T)}]$, where T in the superscript represents the number of data points, and $(\cdot)^\mathrm{T}$ in Eq. (11)

is the transpose. Thus, the annualized APT calculated portfolio error versus a benchmark is

$$\sigma_{P,b} = \sqrt{52(\mathbf{w}_P - \mathbf{w}_b)^T(\mathbf{B}^T\mathbf{B} + \Sigma)(\mathbf{w}_P - \mathbf{w}_b)} \qquad (12)$$

$$\sigma_{ste} = \sqrt{52(\mathbf{w}_P - \mathbf{w}_b)^T\Sigma(\mathbf{w}_P - \mathbf{w}_b)} \qquad (13)$$

and $\sigma_{P,b}^2 - \sigma_{ste}^2$ is the systematic tracking variance of the portfolio, and its square root is the systematic tracking error. The number 52 here represents the average number of weeks in one year.

We can define the portfolio Value-at-Risk (VaR) as the probability that the value of the portfolio will decline, from its current value, V_0, by at least the amount $V(\alpha, T)$, where T is the time horizon, and α is a specified parameter, i.e., $\alpha = 0.05$ Then,

$$\text{Prob}(V_T < V_0 - V(\alpha, T)) \le 0.95 \qquad \text{or}$$
$$\text{Prob}(V_T < V_0 - V(\alpha, T)) \le 0.05, \qquad \text{if } \alpha = 0.95. \qquad (14)$$

The second case indicates that the probability that the value of the portfolio will decline by an amount $V(\alpha, T)$ within 30 days is at most 0.05.

Blin et al. [27] used a 20-factor beta model of covariances based on 3.5 years of weekly stock returns data. The Blin and Bender APT model followed the Ross factor theory, but Blin and Bender estimated at least 20 orthogonal factors. The tradeoff curves in Guerard [24] were created by varying lambda, a measure of risk-aversion, as a portfolio decision variable. Lambda measures the tradeoff between the portfolio expected return and risk. The Markowitz Efficient Frontier can be estimated by estimating a number of risk-aversion levels to estimate the risk-return tradeoff. A higher lambda leads to a higher ratio of portfolio expected return to portfolio risk. As lambda rises, the expected return of the portfolio rises and the number securities in the portfolio declines.

A general stock-selection model for U.S. and global equity markets

In 1991, Markowitz headed the Daiwa Securities Trust Global Portfolio Research Department (GPRD). The Markowitz team estimated stock-selection models using Graham and Dodd [28] fundamental valuation variables, earnings, book value, cash flow and sales, and relative variables, defined as the ratio of the absolute fundamental variable ratios divided by the 60-month averages of the fundamental variables. Bloch et al. [29] reported a set of approximately 200 simulations of U.S. and Japanese equity models. Guerard et al. [7] extended a stock-selection model, originally developed and estimated in Bloch et al. [29] by adding a price momentum (PM) and an analysts' expectations variable. The PM variable is the price in last month divided by the price 12 months prior, and the analysts' expectations variable involves the consensus I/B/E/S (Institutional Brokers' Estimate System) analysts' earnings

forecasts and analysts' revisions composite analysts' efficiency variable, Consensus Temporary Earnings Forecasting (CTEF). Guerard [24] used the CTEF variable that is composed of forecasted earnings yield (FEP), earnings revisions (EREV), and EB (earnings breadth), which represents the direction of revisions and can also be identified as breadth, as created in Guerard et al. [30]. Guerard also reported domestic (U.S.) evidence that the predicted earnings yield is incorporated into the stock price through the earnings yield risk index. Moreover, CTEF dominates the historic low price-to-earnings effect, or high earnings-to-price (EP). The reader is referred to Guerard [30] for a more detailed analysis of the USER model. Fama and French [32–35] presented evidence to support the book-value-to-price (BP) and PM variables as anomalies. Guerard et al. [30] referred to the stock-selection model as a USER model. We can estimate an expanded stock-selection model to use as an input of expected returns in an optimization analysis at time t.

At time $t + 1$, the stock-selection model estimated in this study, denoted as USER, is

$$TR_{t+1} = a_0 + a_1 EP_t + a_2 BP_t + a_3 CP_t + a_4 SP_t + a_5 REP_t$$
$$+ a_6 RBP_t + a_7 RCP_t + a_8 RSP_t + a_9 CTEF_t + a_{10} PM_t + e_t \qquad (15)$$

where

EP	[earnings per share]/[price per share] = earnings − price ratio;
BP	[book value per share]/[price per share] = book value − price ratio;
CP	[cash flow per share]/[price per share] = cash flow − price ratio;
SP	[net sales per share]/[price per share] = sales − price ratio;
REP	[current EP ratio]/[average EP ratio over the past five years];
RBP	[current BP ratio]/[average BP ratio over the past five years];
RCP	[current CP ratio]/[average CP ratio over the past five years];
RSP	[current SP ratio]/[average SP ratio over the past five years];
CTEF	consensus earnings per share I/B/E/S forecast, revisions, and breadth;
PM	price momentum; and
e	randomly distributed error term.

The USER model is estimated using a weighted latent root regression (WLRR) analysis on Eq. (15) to identify variables statistically significant at the 10% level, and the model uses the normalized coefficients as weights and averages the variable weights over the past 12 months. The outlier-adjustment procedure is the Beaton-Tukey biweight procedure implemented and described in Bloch et al. [29].

The 12-month smoothing is consistent with the four-quarter smoothing in Bloch et al. [29]. Although EP and BP variables are significant in explaining returns, the majority of the forecast performance is attributable to other model variables, namely the relative earnings-to-price, relative cash-to-price, relative sales-to-price, PM, and earnings forecast variables. The CTEF and PM variables accounted for 44% of the weights in the USER model.

Extension to mean-ETL optimization model

Economists have been concerned with portfolio shortfall since 1959 when Markowitz addressed the topic in Chapter 9, "The Semi-Variance," in his *Portfolio Selection* [3]. Modern shortfall portfolio construction models, often referred to as "lower partial moments," are found in Bawa and Lindenberg [36], Jensen and King [37], King and Jensen [38], King [39], and Bertsimas et al. [40]. How to choose risk measure in a portfolio construction problem is an important part. The reason to choose ETL as a risk measure to construct a portfolio and compare with the GLER and the USER models is described in the following section.

Value-at-risk (VaR) has been a commonly used risk measure for a long time, which has an intuitive interpretation in practice. Artzner and Delbaen [41] proposed ETL as an alternative risk measure, which satisfies all the axioms of coherent risk measures, monotonicity, positive homogeneity, subadditivity, and the invariance property. In addition, ETL can be called Conditional Value at Risk (CVaR) or Expected Shortfall (ES). Compared with VaR, ETL is convex for all possible portfolios, which means that it always accounts for the diversification effect. ETL is defined as

$$ETL_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha Var_p(X)dp, \qquad (16)$$

where X is the returns, α is the tail probability, and $VaR_\alpha(X)$ is the value-at-risk of X:

$$Var_\alpha(X) = -\inf(x : P(X \leq x) \geq \alpha). \qquad (17)$$

ETL can be regarded as the average loss, or left tail, beyond the VaR threshold, and ETL can also be represented through a minimization formula [42]:

$$ETL_\alpha(X) = \min_{\theta \in \mathbb{R}} \left(\theta + \frac{1}{\alpha} E(-X - \theta)_+ \right), \qquad (18)$$

where $(x)_+ = \max(x, 0)$.

In addition, ETL at tail probability α is one of the spectral risk measures, defined as

$$\rho_\phi(X) = \int_0^1 VaR_p(X)\phi(p)dp, \qquad (19)$$

where $\phi(p)$, $p \in [0, 1]$ is the weighting function, or risk spectrum function. To ensure $\rho_\phi(X)$ to be a coherent risk

measure, the risk spectrum function, $\phi(p)$, should satisfy the following criteria: $\phi(p) \geq 0$, $\phi(p)$ is nonincreasing, and $\int_0^1 \phi(p)dp = 1$. Rockafellar and Uryasev [42] have a more complete description of ETL properties.

In our mean-ETL portfolio optimization, we need to calculate the estimated ETL of scenarios, which are discrete return points, rather than a continuous function. For example, in this case, let $r_1, r_2, \ldots, r_S$ be the S scenario returns of the next time point, and we sort the scenario returns by $r_{(1)} \leq r_{(2)} \leq, \ldots, \leq r_{(n)}$. Then, the ETL_α can be estimated by the following formula:

$$ETL_\alpha = -\frac{1}{\alpha} \left(\frac{1}{S} \sum_{k=1}^{\lceil S\alpha \rceil - 1} r_{(k)} + \left(\alpha - \frac{\lceil S\alpha \rceil - 1}{S} \right) r_{(\lceil S\alpha \rceil)} \right), \quad (20)$$

where $\lceil x \rceil$ means the smallest integer larger than x. Mean-ETL portfolio optimization will use this formula to calculate the estimated portfolio ETL from generated scenarios.

Multivariate normal tempered stable distribution (MNTS) is proposed here to capture the correlation between different securities. We can use independent normal tempered stable (NTS) innovation to model each stock individually. However, one of our targets is to construct a portfolio that can lower the downside risk and have a higher GLER value, and the correlation of GLER between different stocks has to be considered. Compared with NTS, MNTS has one more parameter—the covariance matrix $\boldsymbol{\rho}$ capturing the central dependence structure of the return MNTS distribution. The definition of MNTS of d dimensions can be defined as follows:

$$X = (X_1, X_2, \ldots, X_d) = \boldsymbol{\mu} + \boldsymbol{\beta}(CT - 1) + \boldsymbol{\gamma}\sqrt{CT}\boldsymbol{\varepsilon}, \qquad (21)$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_d)^T$, $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_d)^T$, $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_d)^T$, and $\boldsymbol{\gamma} > 0$.

CT is a classical tempered stable (CTS) subordinator with parameter (α, θ), where $\alpha \in (0, 1)$ and $\theta > 0$. $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_d)^T$ is a d-dim standard normal distribution with covariance matrix $\boldsymbol{\rho}$, independent of the subordinator $CT > 0$. $(\cdot)^T$ is the transpose.

The autoregressive-moving-average (ARMA) generalized autoregressive conditional heteroskedastic (GARCH) and the multivariate normal tempered stable distribution (MNTS), denoted as ARMA-GARCH-MNTS, represent a flexible model for asset returns as it captures volatility clustering, heavy tails (the distributions are Pareto-stable-type except the very extreme tails, where it becomes exponential), skewness in any dimension, and a well-defined equivalent martingale measure. Model parameters can be estimated (via fast Fourier transformation) or calibrated; see Panorska and Mittnik [43]. To generate the scenarios at next time t, we propose the use of ARMA-GARCH-MNTS to model the changes of GLER. Fitting ARMA-GARCH-MNTS involves many parameters to estimate, and it is better to have longer

in-sample data to fit the model. We propose the use of 120 months, or 10 years, of data as in-sample data to estimate the parameters in ARMA-GARCH-MNTS. First, we eliminate the securities that have too many missing points in out-of-sample data by requiring the maximum number of missing months in out-of-sample to be 10. In addition, all of the parameters are estimated for every out-of-sample month. To reduce the number of candidate securities and make use of the robust property of rankings, we select the stocks that have better performance over the last 5 months. Thus, at time t, we will choose the stocks with higher rankings from $t-5$ to $t-1$. We have a rolling window with a fixed length of 120 months to estimate the parameters of the ARMA-GARCH-MNTS model.

For convenience, we use $G_t^{(k)}$ to denote the GLER value of the k_t stock at time t, where $1 \leq t \leq T$ and $1 \leq k \leq N$. T and N are the number of data points and securities, respectively. $G_t^{(k)}$ are positive values because they are the monthly ranking of each stock. To apply ARMA-GARCH-MNTS to fit the data and generate scenarios, we perform the following transformations for security k at time t:

$$
\begin{aligned}
r_t^{(k)} &= f(x) \\
&= \begin{cases} \log\left(G_t^{(k)}\right) - \log\left(G_{t-1}^{(k)}\right), & t \geq 2 \text{ and } k = 1, \dots, N \\ 0, & t = 1 \text{ and } k = 1, \dots, N, \end{cases}
\end{aligned} \tag{22}
$$

where N denotes the number of securities in the portfolio.

We fit the ARMA-GARCH-MNTS model monthly transformed GLER data and then generate $S = 10{,}000$ scenarios of $r_t^{(k)}$. For convenience, we use $\hat{r}_t^{(k,s)}$ to denote the s-th scenario forecasting of k-th stock at time t. We want to maximize the GM of GLER and lower the downside risk. However, our forecasting model is applied to $r_t^{(k)}$, which is indicated in Eq. (22), instead of modeling on GLER directly. We generate the scenarios of $r_t^{(k)}$, and the following transformation is performed to obtain the scenarios of GLER $\widehat{G}_t^{(k,s)}$ from $\hat{r}_t^{(k,s)}$:

$$
\widehat{G}_t^{(k,s)} = \exp\left(\hat{r}_t^{(k,s)} + \log\left(G_{t-1}^{(k)}\right)\right). \tag{23}
$$

Here, $2 \leq t \leq T$, $1 \leq k \leq N$, and $1 \leq s \leq S$. T is the number of data points, N is the number of securities, and S is the number of scenarios.

After generating the scenarios of GLER values, $\widehat{G}_t^{(k,s)}$, we need to perform some transformations to ensure $0 \leq \widehat{G}_t^{(k,s)} \leq 99$. The stocks with the best performance will be 99, whereas the stocks with the worst performance will be 0. The approach we use here is to rank $\widehat{G}_t^{(k,s)}$ for each scenario s and then replace $\widehat{G}_t^{(k,s)}$ by the ranking. By using this approach, we can ensure $0 \leq \widetilde{G}_t^{(k,s)} \leq 99$ while the relative advantages of better securities are staying the same. Then, at time t, we obtain the optimal weights by mean-ETL optimization from the scenarios generated by ARMA-GARCH-MNTS.

Here, mean-ETL optimization tries to maximize a utility function that includes the expected GLER and ETL_α of the scenarios. Thus, at time t the optimization problem can be described as follows:

$$
\max_{\mathbf{w}_t} \frac{1}{S} \sum_{s=1}^{S} \mathbf{w}_t^{\mathrm{T}} \widehat{\mathbf{G}}_t^{(s)} - \lambda ETL_\alpha\left(\mathbf{w}_t^{\mathrm{T}} \widehat{\mathbf{G}}_t\right),
$$

subject to $0.04 \geq \mathbf{w}_t \geq 0$

$$
\mathbf{e}_1^{\mathrm{T}} \mathbf{w}_t = 1 \tag{24}
$$

where $\mathbf{w}_t$ is a column vector of optimal securities weights in the portfolio and $\widehat{\mathbf{G}}_t^{(s)}$ is a column vector that contains the s-th GLER scenario of all N securities:

$$
\widehat{\mathbf{G}}_t^{(s)} = \left[\widehat{G}_t^{(1,s)}, \widehat{G}_t^{(2,s)}, \dots, \widehat{G}_t^{(N,s)}\right]^{\mathrm{T}}
$$

and $\widehat{\mathbf{G}}_t$ should be a matrix that consists of all S scenarios at time t:

$$
\widehat{\mathbf{G}}_t = \left[\widehat{\mathbf{G}}_t^{(1)}, \widehat{\mathbf{G}}_t^{(2)}, \dots, \widehat{\mathbf{G}}_t^{(S)}\right]_{N \times S}.
$$

$(\cdot)^{\mathrm{T}}$ is the transpose. In our portfolio construction, α for ETL is chosen to be 1%, and the risk-averse parameter λ is chosen to be 1.

Portfolio simulation results with the USER and GLER models

Let us briefly review the Guerard et al. [30] Wharton Research Data Services (WRDS) USER model simulation to provide a baseline for comparison with the model in global markets during the 1999 to 2009 period. The portfolio returns of the USER model with APT MVTaR and a lambda of 200 are shown in **Table 1**. We report the Axioma attribution that the USER model produced a 10.7% annual active return, a result consistent with the Barra attribution results reported in Guerard et al. [7]. The active return is derived from a specific return of 16.3% annually. The active return has an Information Ratio of 1.12 and a t statistic of 3.68. Thus, the USER model is effective because of "bottom-up" stock selection. The APT-derived portfolios have active exposures to momentum, value, and size (0.49, 0.43, and -1.05, respectively) that are "priced" by the market to produce statistical significant portfolio factor returns with t statistics of 4.07, 4.24, and 2.9, respectively. The USER model portfolios have positive exposure to value and momentum, and smaller stocks are purchased. Similar factor exposures and statistically significant asset selection were found in the Tsuchida et al. [8] Mean-ETL portfolios using the USER series (Table 1). One can use WRDS Global Compustat data to estimate Eq. (15) weights. The WRDS GLER model, over the 1999 to 2009 period, produces an information ratio of 1.25 ($t = 3.92$) and excess returns of 13.18%, compared with WRDS USER excess returns of 10.7%. Global portfolios outperform the U.S.-only portfolios.

Table 1 Attribution of the USER model using the Axioma U.S. Fundamental Risk Model (USER). (IR: information ratio; HR: hit rate; T-stat: *t* statistics; n/a: not applicable. Adapted from [31].)

Local returns	Return (geometric mean)	Risk (standard deviation)	IR	T-stat	Beginning no. of assets	End no. of assets
Portfolio	0.095	0.221	n/a	n/a	94	92
Benchmark	-0.012	0.221	n/a	n/a	1854	1878
Active	0.107	0.096	1.115	3.683	1898	1922

Contributors to Active Return by Style

Style	Contribution	Average weighted exposure	HR (% success)	Risk	IR	T-stat
Size	0.042	−1.053	0.557	0.047	0.881	2.911
Medium-term momentum	0.026	0.486	0.748	0.021	1.232	4.071
Value	0.010	0.433	0.710	0.008	1.286	4.248
Market sensitivity	0.000	0.063	0.550	0.012	0.019	0.063
Exchange rate sensitivity	0.000	−0.357	0.580	0.007	0.027	0.090
Growth	−0.001	−0.044	0.443	0.002	−0.682	−2.252
Short-term momentum	−0.007	0.055	0.405	0.010	−0.735	−2.428
Leverage	−0.011	0.351	0.458	0.008	−1.298	−4.288
Liquidity	−0.046	−1.148	0.351	0.036	−1.269	−4.194
Volatility	−0.057	0.399	0.244	0.022	−2.591	−8.560

Contributors to Active Return by Sectors

Sectors	Contribution	Average weighted exposure	HR (% success)	Risk	IR	T-stat
Consumer Staples	0.007	−0.065	0.466	0.015	0.481	1.589
Energy	0.005	0.001	0.542	0.007	0.689	2.277
Industrials	0.005	-0.034	0.466	0.016	0.317	1.046
Health care	0.003	-0.032	0.550	0.014	0.240	0.793
Materials	0.002	0.031	0.595	0.011	0.186	0.615
Utilities	0.000	0.015	0.565	0.006	0.041	0.135
Consumer discretionary	−0.007	0.061	0.527	0.023	−0.294	−0.973
Information technology	−0.008	−0.108	0.405	0.035	−0.225	−0.743
Financials	−0.009	0.058	0.466	0.024	−0.360	−1.190
Telecommunication services	−0.010	0.071	0.496	0.016	−0.654	−2.160

In the world of business, one does not access academic databases annually, or even quarterly. Most industry analysis uses the FactSet database and the Thomson Financial I/B/E/S earnings forecasting database. We estimated the result of Eq. (15) for all securities on the Thomson Financial and FactSet databases, some 46,550 firms in December 2011. We estimated the GLER model from the FactSet universe for the 1990–2011 period and used FSGLER (FactSet Global Expected Return) to denote the expected return score. The average estimated GLER weights were as follows: $a_1(0.048)$, $a_2(0.069)$, $a_3(0.044)$, $a_4(0.047)$, $a_5(0.050)$, $a_6(0.032)$, $a_7(0.039)$, $a_8(0.086)$, $a_9(0.216)$, and $a_{10}(0.257)$.

In terms of information coefficients (ICs), the use of the WLRR procedure produces a higher IC for the models during the 1990-2009 time period, .044, versus the equally weighted IC of 0.035. The GLER model, compared with the USER model in Guerard et al. [7], has approximately the same ICs, but a *t* statistic of 6.91, statistically significant at the 5% level compared with the USER model *t* statistic of 5.77, also statistically significant at the 5% level, over the corresponding period. The IC test of statistical significance can be referred to as a Level I test. Further evidence on the anomalies is found in Levy [44]. The USER and GLER simulation conditions are identical to those described in Guerard et al. [7], which used monthly optimization with 8% turnover. Guerard et al.

Table 2 Barra attribution of APT-created FSGLER (global) portfolios, 2003 to 2011. (NoRC: no risk control model; MRC: moderate risk control model; SRC: strong risk control model; ACWG: All Country World Growth index; STD: portfolio standard deviation.)

	Portfolio Performance Criteria			
	NoRC	*MRC*	*SRC*	*ACWG benchmark*
Geometric mean	14.16%	13.75%	11.08%	4.56%
Information ratio	0.65	0.59	0.54	
Sharpe ratio	0.53	0.49	0.41	0.16
Excess returns	9.60%	9.19%	6.52%	
STD	23.20%	24.18%	22.71%	17.15%

Table 3 Attribution of FSGLER APT-created portfolios using the Axioma World Fundamental Risk Model. The term *local* refers to the domestic market.

Source of return	Contribution	Average exposure	Hit rate	Risk	IR	T-stat
Portfolio	12.62%			21.18%		
Benchmark	1.69%			19.59%		
Active	10.93%			8.53%	1.22	3.85
Factor contribution	7.01%			6.67%	1.05	3.31
Style	4.47%			6.24%	0.72	2.26
Exchange rate sensitivity	0.02%	0.014	43.70%	0.26%	0.08	0.27
Growth	0.77%	0.392	68.91%	0.46%	1.65	5.21
Leverage	−0.27%	0.064	47.06%	0.19%	−1.46	−4.59
Liquidity	0.34%	0.177	57.98%	0.64%	0.54	1.69
Medium-term momentum	4.78%	0.486	74.79%	2.17%	2.21	6.95
Short-term momentum	1.69%	0.025	47.90%	1.30%	1.30	4.11
Size	0.86%	−0.746	53.78%	4.57%	0.19	0.60
Value	1.57%	0.153	58.82%	0.91%	1.73	5.44
Volatility	−5.30%	0.501	36.13%	4.16%	−1.27	−4.01
Country	2.25%			2.11%	1.07	3.36
Industry	−0.46%			1.65%	−0.28	−0.88
Currency	0.35%			1.21%	0.29	0.90
Local	0.10%			0.28%	0.36	1.13
Market	0.31%			1.78%	0.17	0.54
Specific return	3.91%			5.95%	0.66	2.07

[7] also used 125 basis points, each way (i.e., for buying and selling stocks), of transactions cost in the United States, and we use 150 basis points, each way, in the global simulation. We restricted the simulations to securities covered by at least two I/B/E/S analysts, a real-world business simulation condition of McKinley Capital Management (MCM). We use the APT risk model and optimizer described in Blin et al. [27] to create portfolios during the 1999-2011 period by varying the portfolio lambda. One seeks to maximize the GM,

Sharpe ratios, and information ratios of portfolios. We experimented with the Wormald and van der Merwe [45] risk control conditions. We find that the No Risk Control (NoRC) condition produced higher information ratios and GMs than the strong risk control (SRC), which is defined as systematic tracking error less than 2.3%, or mild risk control (MRC), which is defined as systematic tracking error less than 1.7%. These results are shown in **Table 2** for the 2003–2011 period. The benchmark is the Morgan Stanley

Table 4 Attribution of FSGLER mean-ETL model using the Axioma World Fundamental Risk Model. The term *style* refers to factor index variables.

Local returns	Return	Risk	IR	T-stat
Portfolio	0.139	0.224		
Benchmark	0.022	0.197		
Active	0.116	0.087	1.33	4.60
Contributor to active returns				
Specific return	0.093	0.042	2.15	7.41
Factor returns	0.026	0.077	0.34	1.18
Contributor to factor returns				
Style	0.029	0.055	0.52	1.80
Country	−0.012	0.040	−0.31	−1.06
Industry	0.018	0.021	0.52	1.81
Currency	−0.32	0.019	−0.17	−0.58
Local	0.030	0.035	0.35	2.95
Market	−0.06	0.038	−0.001	−0.59

Capital International All Country World Growth (MSCI ACWG) index.

We have established that the NoRC model was effective in the FSGLER universe. The portfolio returns of the FSGLER NoRC model with a lambda of 200 are shown in **Table 3**. We report that the GLER model produced a 12.62% annual return, a result consistent with the USER attribution results. The active return of 10.93% is derived from a factor return of 7.01% annually. The active return has an information ratio of 1.22 and a *t* statistic of 3.85. The active return is highly statistically significant.

The GLER model is effective because of stock selection. The APT-derived portfolios have active exposures to momentum, value, and size (0.49, 0.15, and −0.75, respectively), and the momentum and value factors are "priced" by the market to produce statistical significant portfolio factor returns with *t* statistics of 6.95, 5.44, and 0.60, respectively. The benchmark is the Russel Global Growth Index. The GLER model portfolios have positive exposure to value and momentum, and smaller stocks are purchased.

The active return of the mean-ETL model with lambda equal to 1 has an 8.73% annual active return, shown in **Table 4**, which has an information ratio of 1.33 and a *t* statistic of 4.60. The stock-selection model produces a statistically significant active return. The annual active return of the mean-ETL portfolio is larger than the return of the Russell Global Growth benchmark.

The USER and GLER portfolios and the respective attribution analyses report statistically significant active returns based on specific asset selection. APT and mean-ETL optimization techniques, and USER and GLER estimated stock-selection models, produce portfolios

consistent with the maximization of the GM, Sharpe ratio, and information ratio.

Conclusions

We addressed several issues in portfolio construction and management using the Guerard et al. [7] USER data and its extension to a global database and estimation universe. We reported that the Markowitz MV optimization technique, the EIT optimization technique, and tracking error at risk models are appropriate for USER and GLER data. Global portfolios dominate domestic portfolios with regard to the return-to-risk statistics for the 1999-2011 period. The Markowitz approach to portfolio construction and management is 60 years old and remains an integral tool of investment research.

Acknowledgments

References

1. H. M. Markowitz, "Portfolio selection," *J. Financ.*, vol. 7, no. 1, pp. 77–91, Mar. 1952.
2. H. M. Markowitz, "The optimization of a quadratic function subject to linear constraints," *Naval Res. Logist. Quart.*, vol. 3, no. 1/2, pp. 111–133, Mar.–Jun. 1956.
3. H. M. Markowitz, *Portfolio Selection: Efficient Diversification of Investment, Cowles Foundation Monograph no. 16.* New York, NY, USA: Wiley, 1959.

4. H. M. Markowitz, *Mean-Variance Analysis in Portfolio Choice and Capital Markets*. Oxford, U.K.: Blackwell, 1987.

5. B. Solnik, "Why not diversify internationally rather than domestically," *Financ. Anal. J.*, vol. 30, no. 4, pp. 48–54, Jul./Aug. 1974.

6. B. Solnik, *International Investments,* 4th ed. Reading, MA, USA: Addison-Wesley, 2000.

7. J. B. Guerard, Jr., M. N. Gultekin, and G. Xu, "Investing with momentum: The past, present, and future," *J. Invest.*, vol. 21, no. 1, pp. 68–80, Spring 2012.

8. N. Tsuchida, X. Zhou, and S. Rachev, "Mean-ETL portfolio selection under maximum weight and turnover constraints based on fundamental security factors," *J. Invest.*, vol. 21, no. 1, pp. 14–24, Spring 2012.

9. H. A. Latane, "Criteria for choice among risky ventures," *J. Polit. Econ.*, vol. 67, no. 2, pp. 144–155, Apr. 1959.

10. H. M. Markowitz, "Investment in the long run: New evidence for an old rule," *J. Financ.*, vol. 31, no. 5, pp. 1273–1286, Dec. 1976.

11. J. B. Guerard, Jr., M. Takano, and Y. Yamane, "The development of efficient portfolios in Japan with particular emphasis on sales and earnings forecasting," *Ann. Oper. Res.*, vol. 45, no. 1–4, pp. 91–108, Dec. 1993.

12. J. Guerard, Jr., E. Krauklis, and M. Kumar, "Further analysis of efficient portfolios with the USER data," *J. Invest.*, vol. 21, no. 1, pp. 81–88, Spring 2012.

13. W. F. Sharpe, "Capital asset prices: A theory of market equilibrium under conditions of risk," *J. Financ.*, vol. 19, no. 3, pp. 425–442, Sep. 1964.

14. J. Lintner, "The valuation of risk assets on the selection of risky investments in stock portfolios and capital investments," *Rev. Econ. Stat.*, vol. 47, no. 1, pp. 13–37, Feb. 1965.

15. J. Mossin, "Equilibrium in a capital asset market," *Econometrica*, vol. 34, no. 4, pp. 768–783, Oct. 1966.

16. W. F. Sharpe, "A simplified model for portfolio analysis," *Manage. Sci.*, vol. 9, no. 2, pp. 277–293, Jan. 1963.

17. B. Rosenberg, "Extra-market components of covariance in security returns," *J. Financ. Quant. Anal.*, vol. 9, no. 2, pp. 263–274, Mar. 1974.

18. S. A. Ross, "The arbitrage theory of capital asset pricing," *J. Econ. Theory*, vol. 13, no. 3, pp. 341–360, Feb. 1976.

19. S. A. Ross and R. Roll, "An empirical investigation of the arbitrage pricing theory," *J. Financ.*, vol. 35, no. 5, pp. 1071–1103, Dec. 1980.

20. B. Rosenberg and V. Marathe, "Tests of capital asset pricing hypotheses," in *Research in Finance*, H. Levy, Ed. Greenwich, CT, USA: JAI Press, 1979.

21. A. Rudd and H. K. Clasing, *Modern Portfolio Theory: The Principles of Investment Management*. Homewood, IL, USA: Dow-Jones Irwin, 1982.

22. R. Grinold and R. Kahn, *Active Portfolio Management*. New York, NY, USA: McGraw-Hill, 1999.

23. G. Conner and R. A. Korajczyk, "Factor models in portfolio and asset pricing theory," in *The Handbook of Portfolio Construction: Contemporary Applications of Markowitz Techniques*, J. Guerard, Ed. New York, NY, USA: Springer-Verlag, 2010.

24. J. Guerard, "Global earnings forecast efficiency," in *Research in Finance*. Bingley, U.K.: Emerald Group Publishing, 2012, pp. 19–47.

25. *APT Analytics Guide,* Sungard APT, London, U.K., 2005.

26. *APT Analytics Guide,* Sungard APT, London, U.K., 2011.

27. J. M. Blin, S. Bender, and J. B. Guerard, Jr., "Earnings forecasts, revisions and momentum in the estimation of efficient market-neutral japanese and U.S. portfolios," in *Research in Finance*, A. Chen, Ed. Greenwich, CT: JAI Press, Inc., 1997, pp. 93–114.

28. B. Graham and D. Dodd, *Security Analysis: Principles and Technique*. New York, NY, USA: McGraw-Hill, 1934.

29. M. Bloch, J. B. Guerard, Jr., H. M. Markowitz, P. Todd, and G. L. Xu, "A comparison of some aspects of the U.S. and Japanese equity markets," *Jpn. World Econ.*, vol. 5, no. 1, pp. 3–26, May 1993.

30. J. B. Guerard, Jr., M. Gultekin, and B. K. Stone, "The role of fundamental data and analysts' earnings breadth, forecasts, and revisions in the creation of efficient portfolios," in *Research in Finance*, A. Chen, Ed. Greenwich, CT: JAI Press, Inc., 1997, pp. 69–92.

31. J. B. Guerard, Jr., *Introduction to Financial Forecasting in Investment Analysis*. New York, NY, USA: Springer-Verlag, 2013.

32. E. F. Fama and K. R. French, "Cross-sectional variation in expected stock returns," *J. Financ.*, vol. 47, no. 2, pp. 427–465, Jun. 1992.

33. E. F. Fama and K. R. French, "Size and the book-to-market factors in earnings and returns," *J. Financ.*, vol. 50, no. 1, pp. 131–155, Mar. 1995.

34. E. F. Fama and K. R. French, "Multifactor explanations of asset pricing anomalies," *J. Financ.*, vol. 51, no. 1, pp. 55–84, Feb. 1996.

35. E. F. Fama and K. R. French, "Dissecting anomalies," *J. Financ.*, vol. 63, no. 4, pp. 1653–1678, Aug. 2008.

36. V. S. Bawa and E. B. Lindenberg, "Capital market equilibrium in a mean-lower partial moment framework," *J. Financ. Econ.*, vol. 5, no. 2, pp. 255–271, Nov. 1977.

37. D. L. Jensen and A. J. King, "Frontier: A graphical interface for portfolio optimization in a piecewise linear-quadratic risk framework," *IBM Syst. J.*, vol. 31, no. 1, pp. 62–70, Jan. 1992.

38. A. J. King and D. L. Jensen, "Linear-quadratic efficient frontiers for portfolio optimization," *Appl. Stoch. Models Data Anal.*, vol. 8, no. 3, pp. 195–207, Sep. 1992.

39. A. J. King, "Asymmetric risk measures and tracking models for portfolio optimization under uncertainty," *Ann. Oper. Res.*, vol. 45, no. 1–4, pp. 165–177, Dec. 1993.

40. D. Bertsimas, G. J. Lauprete, and A. Samarov, "Shortfall as a risk measure: Properties, optimzation and applications," *J. Econ. Dynam. Control*, vol. 28, no. 7, pp. 1353–1381, Apr. 2004.

41. P. Artzner and F. Delbaen, "Coherent measures of risk," *Math. Financ.*, vol. 9, no. 3, pp. 203–228, Jul. 1999.

42. R. T. Rockafellar and S. Uryasev, "Conditional value-at-risk for general loss distributions," *J. Bank. Financ.*, vol. 26, no. 7, pp. 1443–1471, Jul. 2002.

43. A. Panorska and S. Mittnik, "Stable GARCH models for financial time series," *Appl. Math. Lett.*, vol. 8, no. 5, pp. 33–37, Sep. 1995.

44. H. Levy, *Introduction to Investments,* 2nd ed. Cincinnati, OH, USA: South-Western College Publishing, 1999.

45. L. Wormald and E. van der Merwe, "Constrained optimization for portfolio construction," *J. Invest.*, vol. 21, no. 1, pp. 44–59, Spring 2012.

John B. Guerard, Jr. *McKinley Capital Management, LLC, Anchorage, AK 99503 USA (jguerard@mckinleycapital.com).*
Dr. Guerard is Director of Quantitative Research at McKinley Capital Management, in Anchorage, Alaska. He earned his A.B. degree in economics from Duke University, M.A. degree in economics from the University of Virginia, M.S.I.M. degree from the Georgia Institute of Technology, and Ph.D. degree in finance from the University of Texas, Austin. He co-managed a Japanese equity portfolio with Harry Markowitz at Daiwa Securities Trust Company. Dr. Guerard has published several monographs, including *The Handbook of Financial Modeling* (Probus, 1989, with H.T. Vaught), *Corporate Financial Policy and R&D Management* (Wiley, 2006, second edition), *Quantitative Corporate Finance* (Springer, 2007, with Eli Schwartz), and *The Handbook of Portfolio Construction: Contemporary Applications of Markowitz Techniques* (Springer, 2010). He serves an Associate Editor of the *Journal of Investing* and *The International Journal of Forecasting*.

Svetlozar (Zari) Todorav Rachev *College of Business, Stony Brook University, Stony Brook, NY 11794 USA (svetlozar.rachev@ stonybrook.edu).* Dr. Rachev is Professor of Finance at the College of

Business, Stony Brook University. He is also the Program Director, at the M.S. (Master of Science) Investment Risk Finance Program, Stony Brook University, and Frey Family Foundation Chair of Quantitative Finance, in the Department of Applied Mathematics and Statistics, SUNY-Stony Brook. Dr. Rachev earned his Ph.D. degree in mathematics at Lomonosov University (Moscow), and a Doctor of Science (Habilitation) in physics and mathematics at the Steklov Mathematical Institute. Dr. Rachev has published many models in quantitative finance, including *The Methods of Distances in the Theory of Probability and Statistics* (John Wiley, Finance, 2013, with L. B. Klebanov, S. Stoyanov, and F. Fabozzi), *Financial Models with Levy Processes and Volatility Clustering* (Springer, New York, 2011, with S.T. Kim, M. Bianchi, and F. Fabozzi), *Probability and Statistics for Finance* (John Wiley, Finance, 2010), *Advanced Stochastic Models, Risk Assessment and Portfolio Optimization: The Ideal Risk, Uncertainty, and Performance Measures* (Wiley, 2007, with S. Stoyanov and F. Fabozzi), and *Stable Paretian Models in Finance* (Wiley, New York, 2000, with S. Mittnik). Dr. Rachev's research areas cover non_Gaussian models in mathematical finance, financial econometrics, factor models for asset returns, market and credit risk management, operational risk assessment and forecast, and portfolio optimization and asset liability modeling.

Barret Pengyuan Shao *Applied Mathematics and Statistics, Stony Brook University, Stony Brook, NY 11794 USA (barretshao@ gmail.com)*. Mr. Shao is a Ph.D. candidate in quantitative finance at SUNY Stony Brook University. He earned his B.S. degree in automatic control from Xiamen University and a B.S. degree in mathematical economics from Wangyanan Institute for Studies in Economics (WISE) in China.