

Superconvergent Patch Recovery of Second Derivatives

Abstract

The educational and practical utilization of finite element error estimation techniques are becoming increasingly important in their own right and are also related to the stabilization techniques in various finite element simulations. Most of the stabilization techniques require accurate estimates of the spatial second partial derivatives of the approximate solution. The most common superconvergent patch recovery procedures are based on the work of Zienkiewicz and Zhu for improved gradient recovery. The implementation of those gradient recovery procedures in a general purpose finite element code requires some heuristic choices. The present work reports on experiences in extending such algorithms to obtaining the second derivatives for linear through quartic finite element interpolation families. Many gradient recovery studies assume uniform meshes with constant geometric Jacobians. The computation of the second derivatives involves a matrix containing the products of the Jacobian coefficients and another matrix involving their derivatives (and thus the second derivatives of the geometry mappings). Here particular attention is given to the effects of variable Jacobians on the accuracy of the second derivatives.