

A multiscale method with Robin boundary conditions for the porous media equations

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Abstract

Miscible displacement of one fluid by another in a porous medium has attracted considerable attention in subsurface modeling with emphasis on enhanced oil recovery applications. Here flow instabilities arising when a fluid with higher mobility displaces another fluid with lower mobility is referred to as viscous fingering. The latter has been the topic of major physical and mathematical studies for over half a century. Recently, viscous fingering has been applied for proppant-filled hydraulic fracture propagation to efficiently transport the proppant to the tip of fractures. The governing mathematical system that represents the displacement of the fluid mixtures consists of pressure, velocity, and concentration.

Here we present a novel approach to the simulation of miscible displacement by employing an adaptive enriched Galerkin finite element methods (EG) coupled with entropy residual stabilization for transport. EG is formulated by enriching the conforming continuous Galerkin finite element method (CG) with piecewise constant functions. EG provides locally and globally conservative fluxes, which is crucial for coupled flow and transport problems. Moreover, EG has fewer degrees of freedom in comparison with discontinuous Galerkin (DG) and an efficient flow solver has been derived which allows for higher order schemes. We have shown theoretically and computationally that a robust preconditioner can be achieved if one adds pre- and post smoothings to a block preconditioner involving CG and jumps in the discontinuous piecewise constants. Dynamic adaptive mesh refinement is applied in treating geological material discontinuities.

An additional advantage of EG is that only those subdomains that require local conservation need to be enriched with a treatment of high order non-matching grids. Our high order EG transport system is coupled with an entropy viscosity residual stabilization method introduced in to avoid spurious oscillations near shocks. Instead of using limiters and non-oscillatory reconstructions, this method employs the local residual of an entropy equation to construct the numerical diffusion, which is added as a nonlinear dissipation to the numerical discretization of the system. The amount of numerical diffusion added is proportional to the computed entropy residual. This technique is independent of mesh and order of approximation and has been shown to be efficient and stable in solving many physical problems with CG. Finally we note that it is crucial to have dynamic mesh adaptivity in order to reduce computational costs for large-scale three dimensional applications; both for flow and transport. We employ the entropy residual for dynamic adaptive mesh refinement to capture the moving interface between the miscible fluids. Our computational results indicate that the entropy residual can be used as an efficient a posteriori error indicator.

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