

# ALTERNATING SIGN MATRIX VARIETIES

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ABSTRACT. Matrix Schubert varieties, introduced by Fulton in the '90s, are affine varieties that “live above” Schubert varieties in the complete flag variety. They have many desirable algebro-geometric properties, such as irreducibility, Cohen–Macaulayness, and easily-computed dimension. They also enjoy a close connection with the symmetric groups.

Alternating sign matrix (ASM) varieties, introduced by Weigandt just several years ago, are generalizations of matrix Schubert varieties in two senses: (1) ASM varieties are unions of matrix Schubert varieties and (2) the defining equations of ASM varieties are determined by ASMs, which are generalizations of permutation matrices. ASMs have been important objects of study in enumerative combinatorics since at least the '80s and appear in statistical mechanics as the 6-vertex lattice model. Although ASMs have a robust combinatorial underpinning and although their irreducible components are matrix Schubert varieties, they are nevertheless much more difficult to get a handle on than matrix Schubert varieties themselves. In this talk, we will define ASMs, compare and contrast with matrix Schubert varieties, and state some open problems.