1. (25 pts) State and prove Schwarz’s Lemma.

2. (25 pts) Give the definition for each of the following:
   a. Let \( f \) have an isolated singularity at \( z = a \). Then the residue of \( f \) at \( z = a \) is . . .
   b. Let \( G \) be a region and let \( f : G \rightarrow \mathbb{R} \). Let \( a \in \partial G \) or \( a = \infty \). Then,
      \[
      \limsup_{z \to a} f(z) = \ldots
      \]
   c. The Poisson kernel \( P_r(\theta) = \ldots \)
   d. A set \( F \subset \mathbb{C} \) is normal . . .
   e. A set \( F \subset \mathbb{C} \) is equicontinuous on a set \( E \subset G \) if . . .

3. (25 pts) Show that exactly four of the roots of \( z^5 + 15z + 1 = 0 \) lie in the annulus \( \text{ann}(0, \frac{3}{2}, 2) \).

4. (25 pts) Let \( G \) be a bounded region in \( \mathbb{C} \).
   a. Let \( \{ f_n \} \subset \mathbb{C} \cap A \) and let \( f \in \mathbb{C} \cap \mathbb{C} \cap A \). Suppose that \( f_n \rightarrow f \) uniformly on \( \partial G \). Show that \( f_n \rightarrow f \) in \( \mathbb{C} \cap \mathbb{C} \).
   b. Give an example of a sequence \( \{ g_n \} \subset \mathbb{C} \cap \mathbb{C} \) and a function \( g \in \mathbb{C} \cap \mathbb{C} \) such that \( g_n \rightarrow g \) uniformly on \( \partial G \) but \( g_n \) does not converge to \( g \) in \( \mathbb{C} \cap \mathbb{C} \).