Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

Notation:  
\( \mathbb{C} \) = set of complex numbers  
\( D = \{ z \in \mathbb{C} : |z| < 1 \} \)  
\( \text{RHP} = \{ z \in \mathbb{C} : \text{Re } z > 0 \} \)  
\( A(D) = \{ f : f \text{ is analytic on } D \} \)

1. Find the bilinear transformation \( M \) which maps 1, -1, 0 to 0, 1, \( \infty \) respectively. Describe the images of \( A, B, C, D \) under \( M \) where \( A, B, C, D \) are given in the diagram to the right.

2. Suppose that \( f \) is entire and that \( \lim_{z \to \infty} f(z) \) exists. Show that \( f \) is constant.

3. Let \( G \) be the open sector described in the diagram to the right. Find a one-to-one, onto conformal map \( f \) which maps \( G \) to \( D \) such that \( f(0) = 0 \).

4. Let \( G \) be the region \( \{ D \cap \text{RHP} \} \setminus \{(0, \frac{1}{2})\} \) (see figure to right). Find a one-to-one, onto conformal map \( f \) which maps \( G \) to \( D \) such that \( f(\frac{1}{2}) = 0 \).
5. Evaluate the integral \[ \int_{\gamma} (z + \overline{z}) \, dz \] where \( \gamma(t) = e^{it}, \ 0 \leq t \leq \pi \)

6. Evaluate the integral \[ \int_{\gamma} \frac{dz}{z} \] where \( \gamma(t) = e^{it}, \ 0 \leq t \leq \frac{3\pi}{2} \)

7. Evaluate the integral \[ \int_{\gamma} \frac{e^{-z} - 1}{(1 + z)^2} \, dz \] where \( \gamma(t) = 2e^{it}, \ 0 \leq t \leq 2\pi \)

8. Evaluate the integral \[ \int_{\gamma} \frac{z^2 + 4}{z^2 - 1} \, dz \] where \( \gamma(t) = 1 + e^{it}, \ 0 \leq t \leq 2\pi \)

9. Let \( G = \mathbb{C} \setminus \{ z = k \frac{\pi}{2} : k \text{ odd integer} \} \). Prove the identity that for all \( z \in G \) that

\[ 1 + \tan^2 z = \sec^2 z \]

[Do not prove this using the identity that for all \( z \in \mathbb{C} \) that \( \sin^2 z + \cos^2 z = 1 \).]

10. Let \( f \in \mathbb{A} \) (\( \mathbb{D} \)) such that \( f : \mathbb{D} \rightarrow \mathbb{D} \). Show that for any positive integer \( n \) that \( |f^{(n)}(0)| \leq n! \)

11. State and prove the Fundamental Theorem of Algebra.