Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. Determine the radius of convergence of each of the following series:
   
   a. \( \sum_{n=1}^{\infty} n^2 (2z-i)^n \)
   
   b. \( \sum_{n=1}^{\infty} \frac{z^n}{(3+i^n)^n} \)

2. Let \( G \) be a region in \( \mathbb{C} \) and let \( f : G \to \mathbb{C} \). Prove that if both \( f \) and \( \overline{f} \) are analytic on \( G \), then \( f \) is constant on \( G \).

3. Show that for all complex \( z \) the following hold:
   
   a. \( \cosh^2 z - \sinh^2 z = 1 \)
   
   b. \( \cos 2z = \cos^2 z - \sin^2 z \)

4. Let \( f(z) = z^{1-i} \). Identify and sketch the image of the line segment \((0, i)\) under \( f \).

5. Let \( M \) be the Möbius transformation which maps \( 1+i, 0, 1-i \) to \( \frac{i}{i-1}, \frac{1}{2}, \frac{i}{i+1} \), resp. Find a formula for \( M \) and identify images of the unit quarter discs under \( M \), i.e., the images of \( D_1, D_2, D_3, D_4 \), where the unit quarter disc \( D_j \) is given by \( D_j = Q_j \cap B(0,1) \). See figure.