1. (10 pts) Find the value of $\int_{\Gamma} \frac{dz}{z^2 - 4z}$ where $\Gamma$ is the circle $|z| = 2$ traversed once in the positive direction.

2. (10 pts) Find the value of $\int_{\Gamma} (z^2 - 4z) \, dz$ where $\Gamma = \gamma_1 + \gamma_2$ and $\gamma_1$ is the line segment from 1 to 1+i and $\gamma_2$ is the line segment from 1+i to i.

3. (8 pts) Consider the following eight contours each of which lies in the doubly punctured plane $\mathbb{C} \setminus \{i, -i\}$. Identify which of the eight contours are homotopic to each other in $\mathbb{C} \setminus \{i, -i\}$.

4. (16 pts) Consider the following domains each of which is either the Right Half-Plane (RHP) with a subset of it omitted or the Right Half-Plane (RHP) with an additional set augmented to it. Determine which of these domains are simply connected. (Here $D(z,r)$ denotes the closed disk centered at $z$ of radius $r$.)

a. $\text{RHP} \setminus (0,1)$ b. $\text{RHP} \setminus [1,4)$ c. $\text{RHP} \setminus [1,2]$ d. $\text{RHP} \setminus \text{ann}(1,1,2)$ e. $\text{RHP} \setminus \text{ann}(1,2,3)$ f. $\text{RHP} \setminus D(0,1)$
g. RHP \( \backslash \text{D(1,1)} \)  

h. RHP \( \backslash \text{D(2,1)} \)

5. (6 pts) Explain why \( e^{1+z^2} \) has an anti-derivative on the Right Half-Plane.

6. (20 pts) Find the value of the following integrals, where \( \Gamma \) is the circle \( |z| = 2 \) traversed once in the positive direction.

   a. \[
   \int_{\Gamma} \frac{z e^z}{3z-2} \, dz
   \]

   b. \[
   \int_{\Gamma} \frac{2z+1}{(z+1)^3(z-3)} \, dz
   \]

7. (8 pts) State the convergence properties of the Taylor series for:

   a. \[
   \frac{1}{1+z^2} \text{ at } z = 2
   \]

   b. \[
   e^{-\sin z} \text{ at } z = 2
   \]

8. (8 pts) Find the circle of convergence for the power series \( \sum_{n=0}^{\infty} \frac{(z-i)^n}{2^n} \)

9. (12 pts) For \( f(z) = \sum_{n=0}^{\infty} (n+1)^2 z^n \) find

   a. \( f^{(4)}(0) \)

   b. \[
   \int_{\Gamma} \frac{e^z f(z)}{z^2} \, dz \text{ where } \Gamma \text{ is the circle } |z| = 2
   \]

   traversed once in the positive direction.

10. (12 pts) Find the Laurent series for \( \frac{1}{z-z^2} \) on the annulus:

    a. \( 0 < |z| < 1 \)

    b. \( 0 < |z-1| < 1 \)