Review IV

1. Section 5.2
   a. Approximating area under a curve $y = f(x)$ over the interval $I = [a,b]$
      i. Partition the interval $I$ uniformly into $n$ subintervals of equal width
      ii. Compute the common uniform width $\Delta x = (b-a)/n$ of the subintervals
      iii. Select the right-hand node point $x_k$ for each subinterval
      iv. For each subinterval compute the rectangular area $f(x_k)\Delta x$
      v. Sum up the rectangular areas
   b. Summation notation and properties
      i. Basic rules: Theorem 5.4
      ii. Summation Formulas: Page 334
   c. Representative problems: 4, 8, 9, 11, 15, 24

2. Section 5.3
   a. Riemann sum for $y = f(x)$ over the interval $I = [a,b]$
      i. Partition the interval $I$ into $n$ subintervals $I_k, 1 \leq k \leq n$
      ii. Compute the width of each subinterval, $\Delta x_k = x_k - x_{k-1}$
      iii. Select a node point $x_k^*$ in each subinterval $I_k$
      iv. For each subinterval $I_k$ compute the product $f(x_k^*)\Delta x_k$
      v. Sum up the products
   b. Definition of definite integral: limit process of Riemann sums
   c. Notation $\int_a^b f(x)dx$
   d. Theorem 5.5 If $f$ is continuous on $[a,b]$, then $\int_a^b f(x)dx$ exists.
   e. Area under a curve $y = f(x)$ over the interval $I = [a,b]$ as an integral $\int_a^b f(x)dx$.
   f. Properties of definite integrals: Theorem 5.6
   g. Representative problems: 3, 9, 10, 18, 19, 26a

3. Section 5.4
   a. First Fundamental Theorem of Calculus: Theorem 5.7
   b. Second Fundamental Theorem of Calculus: Theorem 5.8
   c. Representative problems: 6, 12, 18, 26, 28, 32, 35, 38, 41, 43

4. Section 5.5
   a. Examples: 2, 3, 4, 8
   b. Representative problems: 3, 6, 8, 15, 18, 24, 29, 33, 35, 42, 46
5. Section 5.6
   a. Definition of differential equation
   b. Direction fields
   c. First-order, separable differential equations
   d. Exponential growth/decay equation
   e. Orthogonal trajectories
   f. Representative problems: 10, 12, 23, 26, 30, 37, 60, 65

6. Section 5.7
   a. Mean Value Theorem for integrals
   b. Average value of \( y = f(x) \) over the interval \( I = [a,b] \)
   c. Representative problems: 3, 8, 18, 21, 25, 35

7. Section 5.8
   a. Integral approximation by Riemann sum
   b. Integral approximation by Trapezoid Rule
   c. Integral approximation by Simpson’s Rule
   d. Error estimates for Trapezoid Rule and Simpson’s Rule
   e. Representative problems: 3, 7, 12, 13, 19