Continuous Random Variables: Probability Density Functions

MATH 3342
Section 4.1

Continuous Random Variables

• An RV for which *both* of the following are satisfied:

  • The set of possible values consists of either:
    • All numbers in a single interval on the number line OR
    • All numbers in a disjoint union of such intervals

  • No possible value of the RV has positive probability
    • i.e. \( P(X=c) = 0 \) for any possible value of \( c \)
Probability Distributions

• The probability distribution of X says how the total probability of 1 is allocated to the various possible X values.

• Commonly described using:
  • Probability Density Functions (pdfs)
  • Probability Density Curves

Histogram: 10 bins
Histogram: 20 bins

Histogram: 40 bins
Histogram: 80 bins

Histogram: 160 bins
Density Curve

Probability Density Functions

• Let $X$ be a continuous RV.
• The probability density function of $X$ is a function $f(x)$ such that for any two numbers $a$ and $b$ ($a \leq b$):

$$ P(a \leq X \leq b) = \int_{a}^{b} f(x) \, dx $$

• Also called the probability distribution of $X$
Probability Density Functions

- For $f(x)$ to be a true pdf, both of the following must be true:

$$f(x) \geq 0 \text{ for all } x$$

$$\int_{-\infty}^{\infty} f(x) \, dx = 1$$

Are these pdfs?
A Uniform Distribution

The Uniform [0, 1] Distribution

- A continuous RV X is said to have a uniform distribution over the interval [A, B] if the pdf is:

\[
f(x; A, B) = \begin{cases} 
\frac{1}{B - A} & A \leq x \leq B \\
0 & \text{otherwise}
\end{cases}
\]
Example: Reaction Temperature

- The reaction temperature $X$ (in °C) for a certain chemical reaction has a uniform distribution with $A = -5$ and $B = 5$.
- Compute $P(X < 0)$
- Compute $P(-2.5 < X < 2.5)$
- Compute $P(-2 \leq X \leq 3)$

Example: Wind Turbine

- The pdf for $X$, the vibratory stress on a wind turbine blade is given by the following formula:

$$ f(x) = \begin{cases} \frac{x}{100^2} \cdot e^{-x^2/(2 \cdot 100^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases} $$

- $P(X < 200) = ???$
- $P(X > 200) = ???$