Probability Distributions for Discrete Random Variables

MATH 3342
Section 3.2

Discrete Random Variables

- An RV whose possible values either:
  - Constitute a finite set OR
  - Can be listed in an infinite ordered sequence
Probability Distributions

- The probability distribution of \( X \) says how the total probability of 1 is allocated to the various possible \( X \) values.
- Commonly described using:
  - Probability Mass Functions (pmfs)
  - Probability Tables
  - Probability Histograms

Probability Mass Functions

- Also called the probability distribution of a discrete RV
- Defined for every number \( x \) by:
  - \( p(x) = P(X = x) = P(\text{all } s \in S : X(s) = x) \)
## Discrete Probability Table

### pmf Expressed in a Table

<table>
<thead>
<tr>
<th>Value of X</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>...</th>
<th>$x_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>$p_1$</td>
<td>$p_2$</td>
<td>$p_3$</td>
<td>...</td>
<td>$p_k$</td>
</tr>
</tbody>
</table>

### Properties

1. For every $p_i$, $0 \leq p_i \leq 1$
2. $p_1 + p_2 + p_3 + \ldots + p_k = 1$

## Example: AC Units

### Weekly Air Conditioning Units Ordered

<table>
<thead>
<tr>
<th>Units ordered $X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.05</td>
<td>0.15</td>
<td>0.27</td>
<td>0.33</td>
<td>0.13</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Is this a valid probability model?

Construct a probability histogram

- $P(X < 2) = ???$
- $P(X \leq 2) = ???$
- $P(X > 2) = ???$
- $P(2 \leq X \leq 4) = ???$
Example: Flipping a Coin

- Classical example of a Bernoulli RV
- Let $p(0) = P(X = 0) = P(\text{Flip T}) = P(T)$
- Then $p(1) = P(X = 1) = P(\text{Flip H}) = P(H)$
- If the coin is fair, what is $p(0)$? $p(1)$?
- What changes if the coin is weighted to make H more likely?

Parameter of a pmf

- A quantity that can be assigned to any one of a number of values that $p(x)$ depends on.

- The collection of all probability distributions for different values of a parameter is called a family of distributions.
Example: Coin Flipping

- Flip a coin until you flip a head
- Let $p = P(H)$
- Let $X$ be a RV denoting the number of flips needed
- Calculate the formula for $p(x)$, the pmf.
- What is $p(3)$ if $p = 0.5$? If $p = 0.25$?
- If $p = 0.5$, $P(X \leq 3) = ???$

The Cumulative Distribution Function

- Often shortened as cdf
- The cdf $F(x)$ of a discrete RV $X$ with pmf $p(x)$ is defined for every number $x$ by:

$$F(x) = P(X \leq x) = \sum_{y:y \leq x} p(y)$$

- $F(x)$ is the probability that the observed value of $X$ will be at most $x$. 
Example: AC Units

<table>
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<th>Units ordered X</th>
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Construct the cdf for X

Proposition

• For any two numbers a and b,

\[ P(a \leq X \leq b) = F(b) - F(a-) \]

• “a-“ denotes the largest possible X value that is strictly less than a.

• If a and b are integers, then:

\[ P(a \leq X \leq b) = F(b) - F(a-1) \]