Math 5311 Spr 2011 - Homework #3

Due Monday 28 February

1. Gockenbach 6.4.5 and 6.4.6. In both problems, use 4 subintervals.

2. Gockenbach 4.4.6 and 4.4.7. You should be able to modify my Mathematica notebook on Euler’s method and Heun’s method to do improved Euler steps and RK4 steps.

3. Repeat Gockenbach 4.4.6 using Heun’s method and RK4. Verify that the global error in Heun’s method is $O(\Delta t^2)$ and the error in RK4 is $O(\Delta t^4)$.

4. Every SPD matrix $M$ has a Cholesky decomposition $M = LL^T$ where $L$ is a lower triangular matrix.
   
   (a) Use the Cholesky decomposition of $M$ to show that the ODE system
   
   $$Mu' + Ku = f$$
   
   can be transformed into the manifestly symmetric system
   
   $$y' + L^{-1}KL^{-T}y = L^{-1}f$$
   
   (b) Both Mathematica and Matlab have functions to perform Cholesky decompositions. Given
   
   $$M = \frac{1}{30} \begin{pmatrix} 4 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$
   
   and
   
   $$K = 5 \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

   compute $L^{-1}KL^{-T}$.

5. Find the largest timestep $h$ for which Euler’s method is stable when applied to the IVP

   $$y' = -5y$$
   
   $$y(0) = 1.$$