Problem 1. State the following.

(1.) The Well-Ordering Property of \( \mathbb{N} \).

(2.) One version of the Principle of Mathematical Induction.

(3.) The definitions of the terms “Finite Set” and “Infinite Set”.

(4.) The definitions of “Denumerable set” and “Countable set”

(5.) Cantor’s Theorem.

(6.) Let \( \mathbb{P} \) be the set of postive numbers in \( \mathbb{R} \). State the three basic properties of \( \mathbb{P} \).

(7.) The definition of \(|x|\), where \( x \in \mathbb{R} \).

(8.) The Triangle Inequality.

(9.) Let \( A \subseteq \mathbb{R} \) be a nonempty set. State the definitions of “an upper bound for \( A \)” and sup(\( A \)).

(10.) The Completeness Property of \( \mathbb{R} \).

(11.) The Archimedean Property.

(12.) The Characterization Theorem for intervals.

(13.) The Nested Intervals Property.
Problem 2. In each part, decide if the given statement is True or False.

(1.) There is an injection from $\mathbb{N}$ to $\{1, 2, 3, 4\}$.

(2.) If $S$ is an infinite set and $T \subseteq S$, it is possible there is a bijection from $T$ to $S$.

(3.) If $S$ is a finite set and $T \subseteq S$, it is possible there is a bijection from $T$ to $S$.

(4.) If $S$ is a countable set and $T \subseteq S$, then $T$ is countable.

(5.) $\mathbb{N} \times \mathbb{N}$ is countable.

(6.) $\mathbb{Q}$ is uncountable.

(7.) $\mathbb{R}$ is countable.

(8.) If there is a surjection from $\mathbb{N}$ to $S$, then $S$ is countable.

(9.) If $S \subseteq \mathbb{N}$, then $S$ is countable.

(10.) If $a \leq b$ and $c \leq d$ then $a + c \leq b + d$.

(11.) If $a \leq b$ and $c \leq d$ then $a - d \leq b - c$.

(12.) If $a \leq b$ and $c \leq d$ then $a - c \leq b - d$.

(13.) If $a < b$ and both are nonzero, then $1/b < 1/a$.

(14.) If $a \leq b$ and $c \in \mathbb{R}$ then $ca \leq cb$.

Problem 3. Prove by induction that

\[
\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1},
\]

for all $n \in \mathbb{N}$.

Problem 4. Let $x_0 = 1$ and $x_1 = 2$. Let $x_n$ be defined recursively for $n \geq 2$ by

\[
x_{n+1} = \frac{1}{2}(x_n + x_{n-1}).
\]

Use induction to show that $1 \leq x_n \leq 2$ for all $n \in \mathbb{N}$.

Problem 5. What is \(\sup\{1 - 1/n \mid n \in \mathbb{N}\}\)?

Justify your answer in detail.
EXAM

Exam 1
Math 4350-201, Summer II, 2012
July 20, 2012

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You must show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., \( \sqrt{2} \), not 1.414).
- This exam has 5 problems. There are 420 points total.

Good luck!